Navier Stroke Equation (NSE) in the Transport Phenomena (Momentum, Heat, And Mass Transfer Balances) For the Movement of Water in A Horizontal Pipe Form a Pumping Station to A Holding Facility Before Distribution

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Abstract- This study investigate the application of Navier Stokes equation in transport phenomenon, specifically examining water flow in horizontal pipe. The Navier-Stokes equations, which describe the mechanics of fluid motion, primarily control the flow behavior when water flows through horizontal pipes. These equations offer a mathematical model for explaining the motion of incompressible, viscous fluids and are derived from the basic ideas of mass, momentum, and energy conservation. The equations show the forces operating on the fluid, such as viscous forces, pressure gradients, and external body forces (if any, like gravity). Typically, in a horizontal pipe, the considerations are the cylindrical coordinates (r,θ,z), where r is the radial distance, θ is the angular position, and z is the axial direction along the pipe. Velocity profile equation for laminar flow was used with assumed range of radial distance of r values from 0 (Centre) to R (pipe wall) to compute the velocity for each r in which the values obtained are tabulated and a plot of the velocity against radial distance demonstrating the inverse proportionality between them. The pipe radius R was used to calculate the volumetric flow rate Q and the data obtained was used to plot the graph of flowrate against pipe radius, The curve is steep and nonlinear, showcasing the R4 dependence. For a fixed pressure gradient and fluid viscosity, the flow rate increases exponentially as the pipe radius increase. The results highlight the significance of Navier Stokes equation in understanding and predicting fluid flow behavior in transport phenomenon.

I. INTRODUCTION

The understanding and improvement upon fluid systems in engineering requires a comprehension of the field of transport phenomena, which include the study of the transmission of mass, heat, and momentum. (Bird et al., 2007) Chemical reactors, Heating, Ventilation, and Air Conditioning (HVAC) systems, water distribution networks, and petroleum pipelines are just a few of the many industrial and environmental applications that depend on these processes according to Cengel, Y. A., & Cimbala, J. M. (2006). Moreso, the coupled behavior of momentum, heat, and mass is complex and essential to the design, analysis, and functioning of fluid-based systems because in many real life situations, these three elements are not conveyed independently but rather interact and impact one another. (Cengel, Y. A., & Cimbala, J. M. 2006). Therefore, in order to ensure system efficiency, optimize performance, minimize energy consumption, and maintain safety and dependability, it is imperative that these phenomena be studied (White, 2011).

The movement of water through pipelines, which are the foundation of water supply and distribution networks, is one important application of transport phenomena. From pump stations, water is frequently moved over great distances to holding tanks, reservoirs, or other storage facilities. (Romero-Gomez et al,. 2008: Peter, & Bala. 2022) There, it is kept until it is required for a variety of applications, including irrigation, industrial processing, and household supply. (White, 2011). In order to maintain ideal flow rates, minimise frictional losses, regulate heat exchange, and avoid contamination, the dynamic interaction of forces and energy transfers that occurs during the movement of water through such systems must be properly regulated. Designing efficient and long-lasting water distribution systems requires an understanding of the basic ideas underlying these procedures (White, 2016).

The Navier-Stokes equations, which describe the mechanics of fluid motion, primarily control the flow behaviour when water flows through horizontal pipes (NPTEL 2018, Munson et al, 2012: Panton 2013). These equations offer a mathematical model for explaining the motion of incompressible, viscous fluids and are derived from the basic ideas of mass, momentum, and energy conservation. The main instrument for examining and forecasting how fluids would behave in different scenarios is the Navier-Stokes equations, which are the cornerstone of fluid mechanics (Bird et al., 2007). They show the forces operating on the fluid, such as viscous forces, pressure gradients, and external body forces (if any, like gravity). (White, 2016).

These essential processes of mass transfer, heat transfer, and momentum must be taken into account simultaneously likewise for water transport systems. According to Kays, & Crawford (1993) the fluid's velocity and pressure distribution are controlled by momentum transfer, which also establishes the pressure drop and pumping needs throughout the pipe's length. When considering energy dissipation due to friction and heat transfer between the water and the surroundings, factors that can influence both the flow rate and water quality, the role of heat transfer becomes particularly significant. Although mass transfer is frequently linked to the movement of suspended or dissolved materials in the water, it is also essential for maintaining the water's quality and purity as it passes through the system, especially when it comes to contamination or particle deposition (Kays & Crawford 1993).

The necessity of efficiently balancing all three transport phenomena adds to the difficulties of transporting water through horizontal pipes. Munson, et al & (2012) opined that mass balance guarantees that the flow is constant and free of unwanted impurities, while heat balance takes into account the thermal impacts of the pump station's operation and any heat losses to the environment, and momentum balance must account for frictional losses and pressure drop along the pipe. Therefore, solving these coupled equations aids in system design optimisation, guaranteeing the most economical energy use, reducing infrastructure deterioration, and preserving water purity. (Linot, et al, 2022).

In order to investigate these basic transport principles on the water flow through a horizontal pipeline from a pump station to a holding storage facility the system is modelled using the Navier-Stokes equations (Peter, & Bala, 2022). The relationship between the mass, heat, and momentum transfer mechanisms within the system and how it impacts the water transport network's overall performance will be of interest (Prandtl, & Tietjens, 2013). So that by exploring the fluid dynamics in detail and looking at the underlying physical principles, the work could aim to provide insights into pipeline design optimisation, energy resource management, and water quality protection during transportation. The analysis presented in this paper will be useful to improve the efficiency and sustainability of water distribution networks, contributing to better practices in both the planning and operation of such infrastructure. (Romero-Gomez, 2008).

II. NAVIER STROKE EQUATION (NSE)

The Navier-Stokes equation is one of the most crucial formulae in fluid mechanics. The equation refers to a collection of formulas that compute the unknown velocity components (x, y, z) and pressure (p) in fluid dynamics by combining the conservation of mass, momentum, and heat with body force, pressure force, and viscous force (Davidzon., 2017) (White, 2011). It is considered as the fluid mechanics equivalent of Newton's well known second law of motion, these partial differential equations describe the flow of viscous fluids and are expressed as (Fefferman, 2000) (Bird et al., 2007):

$$\rho\left(\frac{\partial v}{\partial t} + v.\nabla v\right) = -\nabla p + \mu\nabla^2 v + f \qquad 1.1$$

Where:
ρ is the density of the fluid,
V is the velocity field,
P is the pressure,

μ is the dynamic viscosity,
f represents external body forces (e.g., gravity),
∇^2 is the Laplacian operator, representing viscous dissipation.

Two scientists, the Irish physicist and mathematician George Gabriel Stokes and the French engineer and physicist Claude Louis Navier developed the equations. Navier first proposed the equations integrating the idea of viscosity into fluid dynamics in 1833, and Stokes improved upon them considerably later, in 1845. The equations take external forces and viscosity into consideration while modelling momentum conservation. Their use in horizontal pipe flow has attracted a lot of interest in the field of transport phenomena because of its usefulness in engineering flow systems (Dumitrescu. et al, 2023).

2.1 FLOW REGIONS IN THE SOLUTION OF THE NAVIER-STOKES EQUATIONS

Even though the Navier-Stokes equations (NSE) are essential to fluid mechanics because they explain how viscous fluids move, they are not always easy to solve because they rely on flow conditions, which differ greatly depending on variables like velocity, viscosity, density, and system size. The flow region, which is the classification of flow behaviour according to the relative importance of various forces (Reynolds Number) acting on the fluid, is defined by these criteria. Consequently, in order to solve the NSE and forecast the fluid's behaviour, it is essential to comprehend and define the proper flow zone.

In addition, the Navier-Stokes equations are generally solved by using certain boundary conditions that are suited to the specific flow regime (laminar, turbulent, or transitional) under study and other simplifying assumptions. The flow region aids in defining the precise form of the equations, and the approaches that would be taken to solve them, as well as the maximum degree of approximation that is possible (Subramanian, 2020).

2.1.1 THE REYNOLDS NUMBER AND FLOW REGIONS

In fluid dynamics, the Reynolds number (Re) is a dimensionless parameter that aids in the prediction of fluid flow patterns in diverse scenarios by evaluating the relationship between inertial and viscous forces (Bird et el, 2007). When operating at low Reynolds numbers, flows typically exhibit dominance of laminar (sheet-like) flow, while at elevated Reynolds numbers, flows tend to manifest turbulence. This turbulence arises from disparities in the velocity and trajectory of the fluid, which can occasionally intersect or even oppose the overall flow direction, generating eddy currents.

The Reynolds number (Re) is defined as mathematically:

$$Re = \frac{\rho U L}{\mu} \qquad 1.2$$

Where:
ρ is the fluid density,
U is the characteristic velocity of the flow,
L is the characteristic length (such as the pipe diameter), and
μ is the dynamic viscosity of the fluid.

The Reynolds number determines the relative importance of inertial forces (which are associated with fluid motion and momentum) to viscous forces (which resist flow and dampen motion) (Bird et el, 2007). An examination of the Reynolds number determines which flow region applies to the system and thereby how to approach the solving of the Navier Stokes equations.

2.1.2 FLOW REGIMES BASED ON REYNOLDS NUMBER

1. Laminar Flow ($Re < 2000$): In the flow pattern, each fluid layer travels parallel to its neighbouring layers with little mixing or disturbance. It is defined by a smooth, ordered fluid layer motion. The flow is usually constant and predictable as the fluid particles follow clearly defined, straight trajectories. Laminar flow occurs at lower velocities, and is frequently found in micro-fluidic systems, with small diameter pipelines, and circumstances where the fluid has a high viscosity; solving the Navier-Stokes equations for laminar flow involves relatively simple techniques (Peter, & Bala, 2022). For example, the velocity distribution in laminar flow within a pipe exhibits a parabolic shape, and either analytical or numerical methodologies can be employed to ascertain solutions to the Navier-Stokes Equations

(NSE). The nonlinear term v.∇v, representing the fluid's convective acceleration, diminishes in importance during the resolution of the NSE for laminar flow, as viscous forces prevail while inertial forces (linked to fluid velocity and acceleration) become negligible.

2. Transitional Flow (2000 < Re < 4000): In this region, both laminar and turbulent flow characteristics are present here therefore the flow is less stable, with even little disturbances that can turn the flow into turbulence (Subramanian, 2020). Depending on variations or disturbances, the system in transitional flow can alternate between laminar and turbulent regimes because of these unstable situations there is the need for extra modelling to account for the development of turbulence, solving the NSE in this region can be a bit more challenging as the instability must be managed, therefore bifherefurcation theory or more intricate numerical simulations that can account for the fluid's propensity to transition between flow modes may be used.
3. Turbulent Flow (Reynolds Number > 4000): At elevated Reynolds numbers, inertial forces predominate, leading to tumultuous, irregular fluid motion. Turbulent flow is characterized by disorderly, erratic fluid movement, featuring eddies and swirls that induce a complex mixing of the fluid. Within this realm, the flow displays high instability, with fluid particles traversing random, unpredictable paths, resulting in fluctuations in velocity and pressure. (Apostol, 2024). Turbulent flow is typically associated with high velocities, large pipe diameters, and low viscosity fluids. In this domain, directly solving the Navier-Stokes Equations (NSE) becomes a formidable task due to the intricate turbulence-induced non-linear interactions that are challenging to capture. To address turbulent flow, engineers often resort to turbulence models (e.g., k-ε models, Reynolds-averaged Navier-Stokes (RANS) equations, or large eddy simulation (LES)), which offer approximations of the turbulent effects.

2.2.1 BOUNDARY CONDITIONS AND FLOW REGIONS

In many fluid flow problems, particularly those involving pipe flow or flow over surfaces, the presence of boundaries affects the fluid flow. The boundary layer is a thin region close to a solid surface where the fluid's velocity changes from zero (because of the no-slip condition at the surface) to the free-stream velocity (because the fluid is far from the surface). (Schlichting, & Gersten, 2016) In all fluid domains, the no-slip condition dictates that the fluid's velocity at solid boundaries is zero relative to the boundary surface.

Symmetry or Free-Surface Conditions also impact boundary layers: In cases of laminar flow, boundary conditions may encompass symmetry (as seen in flow within a pipe, for instance) or prescribed velocity profiles. In turbulent flow scenarios, boundary conditions grow more intricate, particularly in proximity to walls where turbulence models must be employed to approximate the boundary layer's effects. (Sharma, et al 1968).

Depending on the Reynolds number, the flow within the boundary layer can be either laminar or turbulent. The boundary layer is especially crucial for comprehending frictional losses, heat transfer, and mass transfer near surfaces.

Fluid can also be classified according to how they respond to applied shear stress or strain rate, A Newtonian fluid's rate of deformation in response to an applied force is exactly proportional to the force. Under normal circumstances, water and the majority of gases behave as Newtonian fluids. Thus, water provides a classical example of where the application of the NSE would serve as the basis for getting the interaction for the solutions of the momentum heat and mass balances for Newtonian fluid flow in a horizontal pipe flow.

Furthermore, fluids can also be categorized based on their density variations during flow as either incompressible or compressible. Incompressible flow pertains to the movement where the density of the fluid remains consistent throughout the process. This assumption holds true for most liquids, such as water, where alterations in density are minimal.
On the hand, compressible flow, involves the movement where the fluid density undergoes significant changes, as seen in gases or when fluids travel at exceedingly high speeds (approaching or surpassing the speed of sound). Within compressible

flow, deviations in pressure and temperature can lead to notable density fluctuations, necessitating the utilization of specialized equations (like the Euler equations or Navier-Stokes equations for compressible fluids) to accurately depict the flow characteristics (Bird et el, 2007).

The Navier-Stokes equations (NSE) delineate the motion of viscous fluids and serve as the cornerstone of fluid mechanics. Nevertheless, the manner in which these equations are resolved differs considerably for incompressible and compressible fluids due to the distinct assumptions concerning the fluid's density. This differentiation impacts both the mathematical formulation and the approach to finding a solution (Bird et el, 2007).

The fundamental distinctions in solving for Incompressible and Compressible Fluids are as follows:

1. Density: In the realm of incompressible fluids, density remains constant, thereby simplifying the equations. Conversely, in the domain of compressible fluids, density is a variable that necessitates determination, exerting influence on other properties such as pressure and temperature.
2. Energy Equation: In the context of incompressible flow quandaries, the energy equation is typically deemed unnecessary, given the assumption of negligible temperature fluctuations. On the contrary, addressing compressible flow demands the resolution of an energy equation, given the significant alterations in temperature and internal energy.
3. Equations of State: The resolution of compressible fluid predicaments often mandates the utilization of an equation of state to establish the interrelation among pressure, density, and temperature, while incompressible fluids typically obviate this necessity.
4. Boundary Conditions: Analyses of incompressible flow commonly transpire under steady-state circumstances with uncomplicated boundary conditions, whereas compressible flow scenarios can entail intricate boundary conditions due to fluctuations in density and pressure, such as supersonic flows or shock waves.

Furthermore, time-dependence of the fluid's motion flow can also be used to classify flows as steady or unsteady flow:

In steady flow conditions the fluid properties at any given point (such as velocity, pressure, and density) do not change over time. The flow remains constant at each point in the system. The governing NSE for steady, incompressible flow in a pipe corresponds to equation 1.1

The time derivative term in that equation $(\frac{\partial v}{\partial t})$ vanishes, and the equation become simplified. For pipe flow, it reduces further based on the symmetry of the system (Peter and Bala, 2022). For example, in laminar flow through a circular pipe, the velocity distribution is parabolic (Hagen Poiseuille law), and the pressure drop ΔP can be expressed as:

$$\Delta P = \frac{8\mu LQ}{\pi R^4} \qquad 1.3$$

Where:
L is the length of the pipe,
Q is the volumetric flow rate,
R is the radius of the pipe.

In this scenario, the flow remains steady over time, and the velocity profile depends on the pressure drop and the fluid’s viscosity. The solution of the NSE under these conditions provides insights into flow characteristics like velocity distribution and pressure losses due to viscous effects.

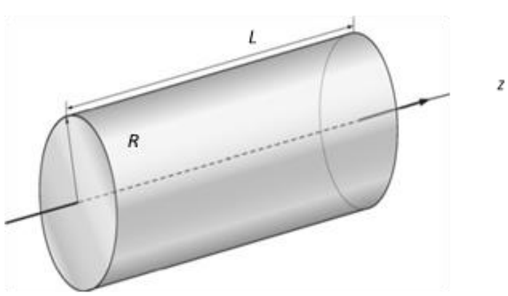
In contrast, unsteady flow refers to flow conditions where the fluid properties at a given point change with time. This could occur due to fluctuating flow rates, varying boundary conditions, or transient phenomena such as pulsating flow in pipes or flow past an object with time-varying conditions.

The solution to this flow pattern depends heavily on the initial and boundary conditions, the flow rate, and external forces. In real-world cases, unsteady flow is commonly observed during the startup of a pump or sudden changes in the demand for water. For instance, during pump operation, pressure fluctuations and velocity variations occur as the system adjusts to the desired flow rate. Therefore, the unsteady Navier-Stokes equations help model the transient behavior of the system, including the time it takes for the system to reach a steady state Numerical methods are used to simulate how pressure and velocity evolve during the transition from unsteady to steady conditions (Peter and Bala, 2020). In the context of water transport from a pump station to a storage tank, both steady and unsteady flow scenarios can occur.

In general, Peter and Bala (2020) contend that the solutions of the NSE are based on simplifying assumptions such as incompressibility or not, steady-state or unsteady state flow, and no-slip boundary conditions at the pipe walls. Although these analytical solutions are only applicable to hypothetical scenarios, they establish essential benchmarks for numerical and experimental validation. Numerical methods are often necessary due to the inherent non-linearity and intricacy of the Navier-Stokes equations (NSE), especially in turbulent and transitional regimes. These equations are commonly tackled using finite element techniques (FEM), finite volume methods (FVM), and finite difference methods (FDM).

Recent advancements include mass-conservative and monotonicity-preserving finite element solvers that enhance simulation accuracy and stability at high Reynolds numbers. Characteristic-based FEM has shown promise in resolving incompressible NSE and three-dimensional transport on unstructured grids, as indicated by computer studies.

III. MATHEMATICAL FORMULATION FOR LAMINAR FLOWS DRIVEN BY PRESSURE GRADIENT



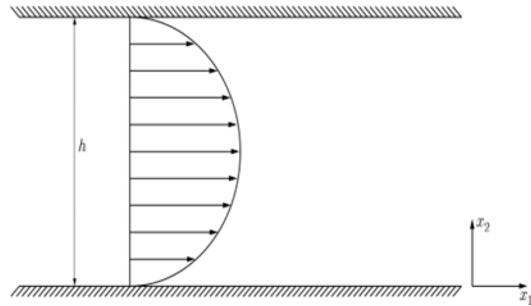


Figure 1 Laminar Flow in a Pipe.

The fig 1 above illustrate the motion of fluid substances in a horizontal pipe the motion of which can be described by the Navier-Stokes equations (NSE). (Fefferman, 2000) (Bird et al., 2007):

The NSE for an incompressible fluid are derived from the principles of conservation of mass and momentum (White, 2011) For an incompressible fluid, the equations in vector form are in the form of equation 1.1 below;

$$\rho\left(\frac{\partial v}{\partial t} + v.\nabla v\right) = -\nabla p + \mu\nabla^2 v + f \quad 1.1$$

where:
$\boldsymbol{\rho}$ is the fluid density (kg/m^3)
$\boldsymbol{v}$ is the velocity vector (m/s)
$\boldsymbol{p}$ is the pressure (N/m^2)
μ is the dynamic viscosity (Ns/m^2)
f are the body forces per unit volume (e.g., gravity). (m/s)

For steady-state flow (no time dependence) and assuming the flow is driven by pressure gradients and viscous forces, the equation simplifies to:

$$\rho(v.\nabla v) = -\nabla p + \mu\nabla^2 \boldsymbol{v} \quad 1.4$$

The continuity equation for an incompressible fluid ensures mass conservation is given by (Bird et el, 2007):

$$\nabla.v = 0 \quad 1.5$$

This equation states that the divergence of the velocity field is zero, meaning the fluid density remains constant over time.

3.1 APPLICATION TO HORIZONTAL PIPE FLOW

Typically, in a horizontal pipe, the considerations are the cylindrical coordinates $(\mathbf{r}, \boldsymbol{\theta}, \mathbf{z})$, where r is the radial distance, $\boldsymbol{\theta}$ is the angular position, and z is the axial direction along the pipe. (Prandtl, & Tietjens, 2013). For simplicity, it is assumed that the fluid is incompressible and flow is symmetric and fully developed, meaning the velocity field depends only on the radial position r and the axial component $\boldsymbol{v_z}$.

In the use of Navier-Stokes equations to solve for flow in a pipe, the boundary conditions for the system must be estimated. The Navier-Stokes equations in cylindrical coordinates will reduce to a form that describes the radial velocity gradient and the pressure gradient driving the flow as shown equation 1.6 for the axial component v_z in a horizontal pipe: (Panton, 2013) and (White, 2016).

Based on assumptions:

1. No radial or angular velocity components: $v_r = 0$, $v_\theta = 0$
2. Steady flow: $\frac{\partial v_z}{\partial t} = 0$
3. Fully developed flow: $\frac{\partial v_z}{\partial z} = 0$
4. Axisymmetric flow: No dependence on θ.
5. Driving force: A constant pressure gradient along z, denoted as $-\frac{\partial P}{\partial z}$.

Therefore, Navier-Stokes equation for $\boldsymbol{v_z}$ simplifies to;

$$\boldsymbol{\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right) = \frac{\partial P}{\partial z}}$$ 1.6

The Boundary Conditions are applied to solve this equation, the following boundary conditions are applied (Schlichting, & Gersten, 2016):

1. No-slip condition at the pipe wall. The fluid velocity at the pipe wall is zero.
   $$\boldsymbol{v_z(r = R) = 0} \quad 1.7$$
2. Symmetry condition at the pipe centerline: The derivative of the velocity with respect to r at the centerline is zero.
   $$\boldsymbol{\frac{\partial v_z}{\partial r}}\,|_{r=0} = \boldsymbol{0} \quad 1.8$$

Velocity Profile: For laminar flow, the velocity profile $v_z(r)$ can be obtained by integrating the simplified Navier-Stokes equation twice. (Bird et el, 2007) The solution is a parabolic velocity profile;

$$v_z(r) = \frac{\Delta P}{4\mu L}(R^2 - r^2) \quad 1.9$$

Where;

ΔP = The pressure-drop across a pipe segment of length L (m)

R = The pipe radius (m)

This profile indicates that the maximum velocity occurs at the center of the pipe and decreases to zero at the pipe walls (Schlichting, & Gersten, 2016).

Flowrate At Laminar Flow

$$Q = \frac{\pi \Delta P R^4}{8\mu L} \quad 1.10$$

This expression highlights the strong dependence of flow rate on pipe radius (R^4), emphasizing the significant impact of small changes in diameter.

3.2 HEAT AND MASS TRANSFER CONSIDERATIONS

3.2.1 Heat Transfer

In addition to momentum transfer, heat transfer may occur due to temperature gradients between the fluid and the surroundings. The energy equation governing heat transfer in the fluid is given by the equation 1.11 (Kays & Crawford 1993):

$$\boldsymbol{\rho Cp\left(\frac{\delta T}{\delta t} + v.\nabla T\right) = k\nabla^2 T} \quad 1.11$$

Where:

Cp is the specific heat capacity (J/KgK)

T is the temperature (K)

k is the thermal conductivity (W/mK)

Likewise, for steady-state conditions and assuming negligible heat generation, the equation simplifies to equation 3.9 (Kays & Crawford 1993):

$$\mathrm{v.\nabla T} = \frac{k}{\rho \mathrm{Cp}}\nabla^2 \mathrm{T} \quad 1.12$$

3.2.1 Mass Transfer

The continuity equation for mass transfer in an incompressible fluid ensures that the mass flow rate remains constant along the pipe (Kays & Crawford 1993) and (Bird et el, 2007).

$$\nabla . v = 0$$ 3.11

This equation is crucial for maintaining the consistency of the fluid flow and ensuring that the volumetric flow rate is conserved.

The mathematical formulation of the Navier-Stokes equations, along with the continuity and energy equations, provides a comprehensive framework for analyzing water flow in horizontal pipes. By understanding the velocity profile, pressure distribution, and heat and mass transfer processes, engineers can design more efficient and effective piping systems.

IV. GRAPHICAL ILLUSTRATIONS

The graphical illustrations serve as a bridge between theory and real-world application. By using properties of Water from the Heat and Mass transfer book for parameters like pressure gradient (G), dynamic viscosity (μ), and pipe radius (R), the graphs provide a visual understanding of:

1. The parabolic velocity profile.
2. The exponential sensitivity of flow rate to radius.

These visualizations are tailored to highlight the intricate dependencies within the system, offering a robust foundation for practical engineering design and analysis.

To solve for a laminar flow system with momentum, heat, and mass transfer, we assume steady state, fully developed flow in a cylindrical pipe.

4.1 PARABOLIC VELOCITY PROFILE (LAMINAR FLOW IN A PIPE)

The parabolic velocity profile in a pipe is derived from the balance of forces acting on the fluid. These forces include the driving pressure gradient, which propels the fluid forward, and the viscous forces, which resist motion and are responsible for the characteristic velocity distribution. In the velocity profile for laminar flow in a horizontal pipe, the velocity is maximum at the centerline and decreases to zero at the pipe wall due to the no-slip condition. This condition, combined with the maximum velocity at the center of the pipe, creates the parabolic shape of the profile.

This framework connects momentum, heat, and mass transfer, optimizing pipeline performance mathematically.

Let's assign values to solve the problem step by step.

- Pipe radius: R = 0.05 m
- Pipe length: L =10 m
- Pressure gradient: $\frac{\partial p}{\partial z} = -100$ Pa/m
- Dynamic viscosity: μ = 0.001 Pa.s
- Thermal conductivity: k = 0.6 W/m.K
- Wall temperature: T_w = 350 K
- Fluid specific heat: cp = 4186 J/kgK
- Fluid density: ρ =1000 kg/m^3
- Diffusion coefficient: D = 1×10−9 m^2/s
- Wall concentration: C_w =1 mol/m^3
- Wall heat flux: q″ = 500 W/m^2
- Wall mass flux: N=0.01 mol/m^2/s

Step 1: Understand the Velocity Equation

For laminar flow in a pipe, the velocity profile is given by:

$$v_z(r) = \frac{1}{4\mu}(-\frac{\partial p}{\partial z})(R^2 - r^2)$$

- $v_z(r)$: Velocity at a radial distance r.
- $\boldsymbol{\mu}$: Dynamic viscosity (0.001 Pa\cdotps for water).
- $\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{z}}$: Pressure gradient (−100 Pa/m).
- $\boldsymbol{R}$: Pipe radius (0.05 m).
- r: Radial distance from the centerline ($0 \le r \le R$).

Step 2: Define Input Values

- Use assumed values:

$$R = 0.05m, \qquad \mu = \frac{0.001Pa}{cdotps}, \qquad \frac{\partial p}{\partial z} = -\frac{100Pa}{m}.$$

- Define a range of r values from 0 (center) to R (pipe wall).

Step 3: Compute Velocity for Each r

For each $\boldsymbol{r}$ value:

1. Substitute $\boldsymbol{r}$, $\boldsymbol{R}$, $\boldsymbol{\mu}$, and $\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{z}}$ into the equation.
2. Calculate $\boldsymbol{v_z(r)}$

For example:

- At $\boldsymbol{r = 0}$ (centerline):
  $v_z(0) = \frac{1}{4\times 0.001}(100)(0.05^2 - 0^2) = 3.125m/s$
- At $r = R = 0.05$ (pipe wall):
  $v_z(0.05) = \frac{1}{4\times 0.001}(100)(0.05^2 - 0.05^2) = 0m/s$

Step 4: Create a Range of r Values and Tabulate

Let us consider water flowing through a horizontal cylindrical pipe with arbitrary data points of radial distance, $R(m)$. Use six equally spaced points between 0 and R to ensure smooth plotting (0.00, 0.01, 0.02, 0.03, 0.04, and 0.05).

- At $r = 0.00$,

$$\boldsymbol{v_z(0) = \frac{1}{4\times 0.001}(100)(0.05^2 - 0^2) = 62.5m/s}$$

- At $r = 0.01$,

$$v_z(0.01) = \frac{1}{4\times 0.001}(100)(0.05^2 - 0.01^2) = 60.0m/s$$

- At $\boldsymbol{r} = 0.02$,

$$v_z(0.02) = \frac{1}{4\times 0.001}(100)(0.05^2 - 0.02^2) = 52.5m/s$$

- At $r = 0.03$,

$$v_z(0.03) = \frac{1}{4\times 0.001}(100)(0.05^2 - 0.03^2) = 40.0m/s$$

- At $r = 0.04$,

$$\boldsymbol{v_z(0.04) = \frac{1}{4\times 0.001}(100)(0.05^2 - 0.04^2) = 22.5m/s}$$

- At $\boldsymbol{r} = 0.05$,

$$\boldsymbol{v_z(0.05) = \frac{1}{4\times 0.001}(100)(0.05^2 - 0.05^2) = 0.0m/s}$$

Table 1 Showing the velocity at various points in the pipeline.

$r(m)$	$v_z(m/s)$
0.00	62.50
0.01	60.00
0.02	52.50
0.03	40.00
0.04	22.50
0.05	0.00

Step 5: Plot the Results

- Use $\boldsymbol{r}$ values on the x-axis (radial distance).
- Use $\boldsymbol{v_z(r)}$ values on the y-axis (velocity).

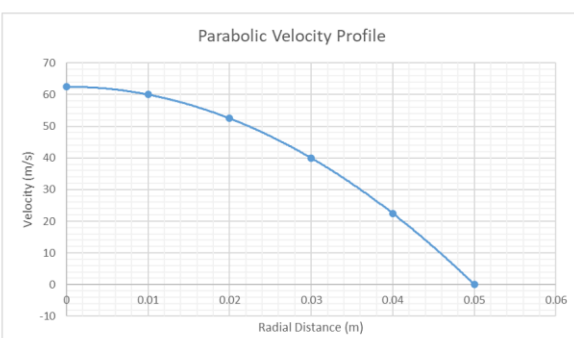


Figure 2 Velocity Profile (Laminar Flow in a Pipe).

4.2 EXPONENTIAL SENSITIVITY OF FLOWRATE TO RADIUS

The graphical illustration of volumetric flow rate as a function of pipe radius offers a comprehensive visualization of how flow rate scales with the size of the pipe under laminar flow conditions. This relationship, derived from the Hagen-Poiseuille equation, emphasizes the strong dependence of flow rate on the fourth power of the pipe radius, making it a critical factor in fluid transport systems. Plotting this graph provides valuable insights into the efficiency of fluid delivery, highlighting how small changes in pipe diameter can result in significant variations in flow rate, thus aiding in the design and optimization of piping systems.

Consider a horizontal cylindrical pipe with arbitrary data points of Pipe Radius $R(m)$, the corresponding values of the volumetric flowrate, Q_z from Equation 3.8 are given in the table below:

$R(m)$	0.010	0.015	0.020	0.025	0.030
$Q_z(m^3/s)$	0.000	0.000	0.001	0.003	0.006

Therefore, graph below shows;

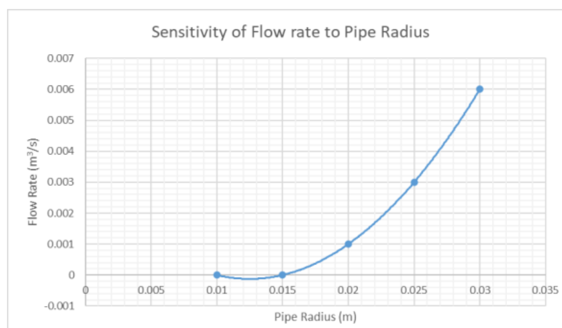


Figure 1 Volumetric Flow Rate as a Function of Pipe Radius.

The curve is steep and nonlinear, showcasing the R^4 dependence. For a fixed pressure gradient and fluid viscosity, the flow rate increases exponentially as the pipe radius increases.

4.3 OBSERVATIONS:

1. Non-linear Growth: The relationship between flow rate and radius is not proportional but follows a fourth-power dependency. This results in rapid

growth in flow rate for even modest increases in radius.

2. Dominance of Radius: Among all parameters in the Hagen-Poiseuille equation, the radius has the most pronounced effect on Q. Changes in other parameters, such as viscosity or pressure gradient, affect Q linearly or inversely, but the exponential sensitivity to R is unparalleled.
3. Zero Flow at Zero Radius: At $R = 0$, the flow rate naturally becomes zero, aligning with the physical reality of no flow in a completely closed or nonexistent pipe.

The graph is essential for designing water distribution networks, oil pipelines, and chemical process systems. It informs decisions on pipe diameter for achieving desired flow rates under specific operating conditions.

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