Application of Navier-Stokes Equation to Study the Flow of Oil Film on a Vertical Wall

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Abstract- This study presents a comprehensive review of analytical approaches to solving the Navier-Stokes equation for steady, incompressible, parallel, laminar flow of oil falling gently down an infinite vertical wall moving with a velocity. The Navier-Stokes equation is a nonlinear partial differential equation that describes the motion of fluids. The study also presents some recent developments in methods for solving problems involving fluid flow by using velocity profiles and representing them in a form of diagram to show the fluid flow using the Navier-Stokes equation.

Indexed Terms- Navier-Stoke equation (NSE), laminar flow, incompressible fluid, oil film, vertical wall, velocity profile.

I. INTRODUCTION

The origin of Navier- stokes equations (NSE) begins with the 1822 paper of C.L.M.H Navier (Ann.Chim.phys. 19, 234-245) who derived equations for homogenous in compressible fluids from a molecular point of view. The continuous derivation of the (NSE) is due to J.C. Saint- Venant (1843) and G.G. stokes (Trans.cambridge philos.soc.1845, 8, 287-319). The (NSE) are generally considered as the universal basis of fluid mechanics and transport phenomena, no matter how complicated and unpredictable the behavior of its solutions may be. The (NSE) is a fundamental equation in fluid dynamics that describes the motion of fluids. The equation is nonlinear and coupled, making it challenging to solve analytically. Numerical methods, such as finite difference, finite element, and finite volume methods, are widely used to solve (NSE), with the application of Newton's second law to the fluid motion together with the assumption that the fluid stress is the sum of a diffusion viscous term (proportional to the gradient of velocity) plus pressure term. Many research journals and books has explained extensively the applications of (NSE) of various degree. Among others see the literatures: Adanhoume et. al. (2015), studied analytical solution for Navier-stokes equations in the cylindrical coordinates. Adigun and Adeniyan (2015), gives solution of hydro magnetic flow and heat, transfer past on exponentially stretching permeable vertical heating effect. Haoxiang and Thomas (2004), considered the contra variant form of the (NSE) in time dependent curvilinear coordinate systems. Kaurangini and Jha (2010), generalized couchette flow in a composite parallel plates channel. Kyle et. al. (2015), use a volume of fluid method for simulating fluid/fluid interfaces in contact with solid boundaries. Peyret and Tylor (2011), generalized mathematical model for the aqueous humors flow driven by temperature gradient. Sonian (2001), study computational methods for fluids flows. Mbah et. al. (2015), use equation of motion for viscous fluids.

The goal of this paper is to show how the Navier – Stokes equations are used to obtain the velocity profile for a fluid flow on a vertical wall.

II. RESEARCH METHODOLOGY

The following steps were applied to solve the flow problem of oil film flow on a vertical wall with the aim of showing the velocity profile of the flow:
STEP 1: Determine the flow geometry and flow domain.
STEP 2: Determine the assumptions, approximations and boundary conditions.
STEP 3: Determine the appropriate differential equations and unknowns and simplify.
STEP 4: Solve the equation.
STEP 5: Apply the boundary conditions in step 2.

III. RESULTS ANALYSIS

Problem (Oil Film on a Vertical Wall)
Consider a steady, incompressible, parallel, laminar flow of oil falling gently down an infinite vertical wall as shown in figure 1. The gravity acts in the negative z direction which is downward as shown in the figure and the oil falls due to gravity effect alone as no pressure is applied to it.

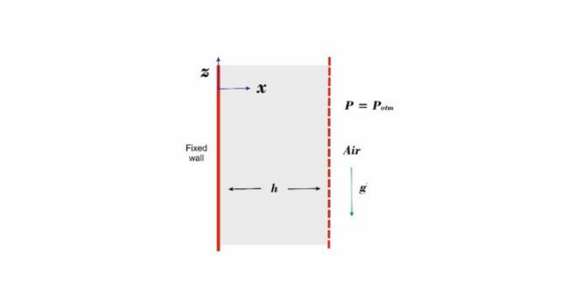

Fig. 1: Oil film on a vertical wall.

Step 1: Consider the oil film on a vertical wall in fig 1 above. Thus let;
z- axis be parallel to the pipe
x- axis be perpendicular to the z-axis
h- oil film thickness
g- gravity
ρ- oil density
μ- oil viscosity

Step 2: The following are the assumptions and approximations;
1. The wall is infinite in the y-z plane (flow is fully developed)
2. The flow is steady, therefore $\left(\frac{\partial}{\partial t} = 0\right)$
3. The flow is parallel $(u_x = 0)$
4. The fluid is incompressible and Newtonian, and the flow is laminar.
5. Pressure $P = P_{atm}$= constant at the free surface since there is no pressure gradient pushing the flow.
6. The velocity field is purely two-dimensional, which implies that $u_y = 0$ and all partial derivative with respect to y are zero.
7. The components of gravity are $\left(g_x = g_y = 0 \; and \; g_z = -g\right)$.

Consider the boundary conditions;
1. At wall, no slip which implies u_x=u_y=u_z=0 at x=0
2. Free surface which implies $\frac{\partial w}{\partial x} = 0$ at x=h
3. At interface $\mu_{oil} \gg \mu_{air}$
Step 3: Using Navier-stoke equation (Conservation of momentum);

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0 \quad \text{Continuity equation} \left(\frac{\partial u_z}{\partial z} = 0\right) \qquad (1)$$

For the x- axis

$$\rho\left(\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z}\right) = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}\right) \qquad (2)$$

For the y-axis

$$\rho\left(\frac{\partial u_y}{\partial t} + u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y} + u_z\frac{\partial u_y}{\partial z}\right) = -\frac{\partial P}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2}\right) \qquad (3)$$

For z-axis

$$\rho\left(\frac{\partial u_z}{\partial t} + u_x\frac{\partial u_z}{\partial x} + u_y\frac{\partial u_z}{\partial y} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right) \qquad (4)$$

Where
ρ = density of fluid (assumed constant)
μ = viscosity of fluid (assumed constant)
g_x = gravity acting on the fluid in the x- direction
g_y = gravity acting on the fluid in the y- direction
g_z = gravity acting on the fluid in the z- direction

Step 4: Solving the equation by applying the assumptions and boundary conditions;
At steady state
$$\frac{\partial u_x}{\partial t} = \frac{\partial u_y}{\partial t} = \frac{\partial u_z}{\partial t} = 0$$
Since the flow does not depend on pressure gradient
$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = \frac{\partial P}{\partial z} = 0$$
Since velocity is in the z direction only, we have
$$u_x = u_y = 0$$
From equation (4), taking into consideration that $g_z = -g$
$$\frac{\partial^2 u_z}{\partial x^2} = \frac{\rho g}{\mu}$$

$$\int \frac{\partial^2 u_z}{\partial x^2} = \int \frac{\rho g}{\mu} dx$$
$$\frac{\partial u_z}{\partial x} = \frac{\rho g}{\mu} x + c_1 \qquad (5)$$

$$\int \frac{\partial u_z}{\partial x} = \int \left(\frac{\rho g}{\mu} x + c_1\right) dx$$
$$u_z = \frac{\rho g}{2\mu} x^2 + c_1 x + c_2 \qquad (6)$$

Step 5: Applying the boundary conditions;
$$At\ x = 0,\ u_z = 0 \ \rightarrow \ 0 = 0 + 0 + c_2 \ \rightarrow \ c_2 = 0$$
$$At\ x = h,\ \frac{\partial u_z}{\partial x} = 0$$
From equation (5)

$c_1 = -\frac{\rho g h}{\mu}$

From equation (6)

$$u_z = \frac{\rho g x}{2\mu}(x - 2h) \quad (7)$$

IV. DISCUSSION OF RESULTS

In the problem, we considered a steady, incompressible, parallel, laminar flow of a film of oil slowly down an infinite vertical wall and found that the velocity profile was given by the equation

$$u_z = \frac{\rho g x}{2\mu}(x - 2h)$$

Which is a half parabola and thus the flow will look like the following diagram

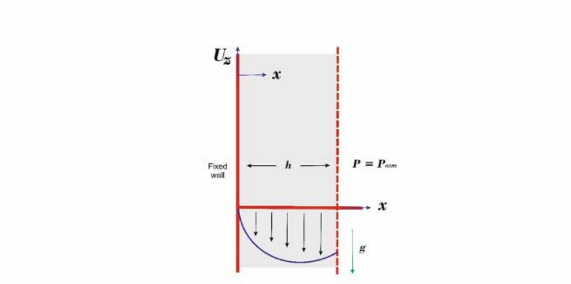

Fig. 2: Velocity profile of oil film through vertical wall.

Interpretation: The velocity profile from figure 2 shows that for the flow problem i.e laminar flow of oil film through a vertical wall, the flow moves in a half-parabolic shape the beside the vertical wall.

CONCLUSION

The velocity profiles for steady, incompressible, parallel, laminar flow of oil falling gently down an infinite vertical wall moving with a velocity was discussed and the result was were obtained. It was shown that the flow moves in a half-parabolic shape beside the vertical while for the. Since our approach and the results obtained in this study are not the same from the results obtained in the literature, which implies that the result of this paper is essentially new.

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