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# Analysis Study of Boundary Layer of Air Flowing on A Flat Plate

OLUGBENGA ADEOLA GRACEA[1], ADISA USMAN OLANREWAJUB[2], MICHEAL JEREMIAHC[3], ADI MELODY GODID[4], THOMAS MELODY OZURUONYEE[5]
*[1, 2, 3, 4, 5]Department of Chemical Engineering, Faculty of Engineering, University of Abuja, Nigeria.*

***Abstract-** This study presents an in-depth investigation of the boundary layer characteristics of air flowing over a flat plate, with a particular emphasis on the transition from laminar to turbulent flow. A detailed experimental analysis was conducted using a wind tunnel, and the results were compared with theoretical models to validate the findings. The experimental setup consisted of a flat plate positioned horizontally in the test section of the wind tunnel. The free-stream velocity was measured using a pitot tube, and the velocity profile was obtained using a hot-wire anemometer. The boundary layer thickness, displacement thickness, momentum thickness, and shape factor were calculated from the experimental data. The results show that the boundary layer thickness increases with distance from the leading edge of the flat plate, while the friction coefficient decreases with increasing velocity. The shape factor remains relatively constant, indicating a consistent boundary layer shape and structure. The velocity profiles exhibit a clear transition from laminar to turbulent flow, with the turbulent boundary layer showing a significant increase in velocity. The theoretical analysis was based on the Blasius solution for laminar flow and the turbulent flow model. The results of the theoretical analysis were compared with the experimental data, and good agreement was found. The study demonstrates the importance of considering both laminar and turbulent flow regimes in the analysis of boundary layer behavior. The findings of this research have significant implications for the design of aerodynamic systems, such as aircraft wings and wind turbines. The results provide valuable insights into the behavior of boundary layers and can be used to optimize the design of these systems for improved performance and efficiency.*

***Indexed Terms-** Boundary layer, Flat plate, Laminar flow, Turbulent flow, Wind tunnel, Aerodynamics, Fluid dynamics.*

## I. INTRODUCTION

The concept of boundary layers were first introduced by Ludwig Prandtl in 1904, who proposed the concept of a thin layer near a solid surface where the fluid's velocity changes rapidly. Since then, numerous studies have investigated the characteristics and behavior of boundary layers in various fluid flow scenarios.

Theoretical models, such as the Blasius solution for laminar flow and the turbulent flow model, have been developed to predict the behavior of boundary layers. Experimental studies have also been conducted to investigate the boundary layer characteristics in various flow regimes.

Research has shown that boundary layers play a crucial role in determining the drag and lift forces experienced by objects in fluid flow. The shape and size of the boundary layer can significantly affect the overall performance of the system.

In addition, boundary layers are involved in heat transfer and mass transport phenomena, making them essential in various engineering applications, such as aerospace, chemical, and mechanical engineering.

Despite the significant progress made in understanding boundary layers, there is still a need for further research to investigate the complex behavior of boundary layers in various flow regimes. This study aims to contribute to the existing body of knowledge by investigating the boundary layer characteristics of air flowing over a flat plate. based on observations of velocity profile in the vicinity of a plate. It describes the region of fluid near a surface where the effects of viscosity are significant. when considering the flow of a fluid past any object, friction plays an important role. As air passes over the surface, the flow adheres to the surface due to friction between the air and solid material of the plate [1]. The flow velocity is zero at the surface, the no-slip conditions, and therefore near the surface there is a region in which the flow is retarded. This region of the flow that is retarded is called the boundary layer [2]. Dimensionally, the boundary layer is described by the boundary layer thickness. It is the distance from the plate to the point where the flow speed is 99% of the outer flow velocity. A velocity profile, which shows the variation of flow speed with the vertical distance from the plate is used to describe the boundary layer. There are two types of boundary layers: laminar and turbulent. The type of boundary layer that will occur depends upon the Reynolds number as well as the surface conditions. The different boundary types have different profiles and different growth rates. Boundary layer theory playing a major rule in aerodynamics. The concept of boundary layers is a fundamental aspect of fluid dynamics, playing a crucial role in understanding the behavior of fluids in various engineering applications. A boundary layer is the thin region near a solid surface where the fluid's velocity, temperature, and concentration gradients are significant.

## II. BOUNDARY LAYER DEVELOPMENT

The boundary layer is a thin region adjacent to a solid surface where the flow velocity transitions from zero (due to the no-slip condition) to the free-stream velocity.

### 2.1 STAGES OF DEVELOPMENT

Laminar boundary layer: characterized by smooth, orderly flow. The criterion for a flow over a flat plate to be laminar is that a dimensionless quantity called Reynolds number,
Re u ' and defined as:

$$Re_x = \frac{\rho U x}{\mu}$$

should be less than $5X10^5$ . The number is the ratio of the inertia forces to the
viscous forces.

Transition region: A region where instability develops, leading to turbulence.

Turbulence boundary layer: characterized a chaotic, mixing flow structure.

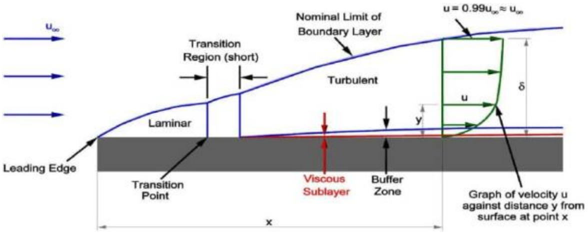

Fig 1; boundary layer on a flat plate.

Factors influencing development

- Surface roughness
- Reynolds number:

$$Re = \frac{\rho U x}{\mu}$$

Where;
$\rho$: density of air
x : distance from the plate
$\mu$ : dynamic viscosity
$U$ : velocity at a point.

### 2.2. FREE STREAM VELOCITY ($U_\infty$) AND BOUNDARY LAYER VELOCITY

Free stream velocity refers to the undisturbed fluid velocity far away from the surface of the flat plate, essentially the velocity of the air (fluid) before it is affected by presence of an object, while boundary layer velocity refers to the velocity of the fluid within a thin layer close to the surface of the flat plate surface, where the fluid is significantly slowed down due to friction with the surface. The boundary layer velocity gradually increases from zero at the surface to reach the free stream velocity at the edge of the boundary layer.

Free-steam velocity equation
A formula to calculate the free stream velocity in fluid dynamics, particularly for airflow is derived from Bernou

$$U_\infty = \sqrt{\frac{2(P_t - P_s)}{\rho}}$$

Where:

- $U_\infty$ = free stream velocity (m/s)

- $P_t$ : total pressure measured using a pitot tube
- $P_s$: static pressure measured using a static port.
- $\rho$: air density (kg/m³), which can be calculated using the ideal gas law:

$$\rho = \frac{P}{RT}$$

Boundary layer equation.

For the velocity profile of a Laminar boundary layer over a flat plate (based on the Blasius solution) is:

$$\frac{u}{U_\infty} = \frac{y}{\delta(x)}$$

Where:

- u = velocity at a point y within the boundary layer.
- $U_\infty =$ free-stream velocity
- $y =$ distance from the plate
- $\delta(x) =$ boundary layer thickness at distance x from the leading edge.

## 2.3. GOVERNING EQUATION OF THE BOUNDARY LAYER

The boundary layer equation is derived from Navier-stokes equations under the assumption that flow is steady, incompressible and two dimensional. these assumptions simplify the analysis near the flat plate.

Simplified governing equations

1. Continuity equation (mass conservation):

$$\boldsymbol{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}}=0$$

2. Momentum equation:

Using Navier-stokes equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\boldsymbol{\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + v\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

Assumptions

1. The flow is assumed to be laminar, meaning its smooth and continuous, without turbulence or chaotic behavior and the velocity within the boundary layer in the x-direction is far greater than the y-direction(U>>V)
2. The fluid is assumed to be incompressible, meaning its density remains constant throughout the flow.
3. The flow is assumed to be two-dimensional meaning it varies in two dimensions (x and y) and not in the third-dimension z. ($\partial z = 0$)
4. The flow is assumed to be steady, meaning it doesn't change over time. ( $\partial t = 0$)
5. No slip condition: the fluid is assumed to have zero velocity at the surface of the flat plate.
6. Constant viscosity throughout the flow.

Gravity effects are assumed to be negligible, or flow are assumed to be horizontal.

Boundary conditions:

@ y=0, u=v=0

@ y= δ, u= $\boldsymbol{U}_\infty$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + v\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

Resulting deferential equation.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + v\frac{\partial^2 u}{\partial y^2}$$

- U: velocity in the x-direction.
- V: velocity in the y-direction.
- $\boldsymbol{\rho}$: *fluid density*
- *V*: kinematic viscosity (v=˹μ/ $\boldsymbol{\rho}$)

## 2.4. Boundary Layer Thickness

Boundary layer thickness is the distance from a solid (flat plate) surface to the point where the velocity of the fluid reaches a certain percentage of the free-stream velocity. It is a thin layer of fluid that forms due to friction between the surface and the fluid.

Laminar flow:

[1]for laminar flow over a smooth flat, the exact (or Blasius) numerical solution gives an expression for boundary layer thickness ($\delta$) along the plate in the

flow direction, at allocation x from the leading edge, as follows:

$$\delta_{hx} = 5xRe_x^{-0.5}\ For\ Re \le 5X10^5$$ [2]

For Turbulent Flow:
In turbulent flow, the thickness grows more rapidly.

$$\boldsymbol{\delta_{hx} = 0.381xRe_x^{-0.2}}\ For\ \ 5X10^5 > \ \mathrm{Re}_x < 10^7$$ [3]

### 2.5. Displacement Thickness

The displacement thickness ($\boldsymbol{\delta_x}$) is the distance by which the boundary layer displaces the free-stream flow.

Laminar flow
for a laminar flat plate, the numerical (or Blasius) solution gives an expression for
displacement thickness ($\boldsymbol{\delta_x}$), as function, which is:

$$\boldsymbol{\delta_x} = \frac{\boldsymbol{\delta_{hx}}}{\mathbf{3}}$$ [4]

For Turbulent Flow:

$$\boldsymbol{\delta_x} = \frac{\boldsymbol{\delta_{hx}}}{\mathbf{8}}$$ [5]

### 2.6. Momentum Thickness

The momentum thickness ($\boldsymbol{\delta_{ix}}$) is the distance from the solid surface (flat plate) to the point where the momentum of the fluid is equal to the momentum of the free stream flow. Based on the exact (or Blasius) numerical solution the expression of the momentum thickness ($\boldsymbol{\delta_{ix}}$) along the plate in the flow direction, at allocation x from the leading edge, as follows:
for a laminar Flow:

$$\boldsymbol{\delta_{ix}} = \frac{\boldsymbol{\delta_{hx}}}{\mathbf{7}}$$ [6]

For Turbulent Flow:

$$\boldsymbol{\delta_{ix}} = (\frac{\mathbf{7}}{\mathbf{72}})\boldsymbol{\delta_{hx}}$$ [7]

### 2.7. Thermal boundary layer thickness

Thermal boundary layer thickness ($\boldsymbol{\delta_{Tx}}$) is the distance from the solid surface (flat plate) to the point where the temperature of the fluid (air) is equal to the free stream temperature. [8]
for a laminar Flow:

$$\delta_{Tx} = \delta_{hx}Pr^{-0.333}$$

For Turbulent Flow:

$$\delta_{Tx} = \delta_{hx}$$

### 2.8. Surface shear stress ($\tau_w$)

shear stress for flat plate in fluid flow is defined as the tangential force per unit area exerted by the fluid on the surface of the plate due to the fluid's viscosity. This stress arises because of the velocity gradient in the boundary layer, where the fluid adheres to the plate (no-slip condition) and the velocity increases.

For laminar zone:
[9]For the laminar zone of flow over a flat plate, the shear stress at the surface ($\tau_w$) can be calculated using.

$$\tau_w = 0.332\rho U_\infty{}^2\frac{1}{\sqrt{Re_x}}$$

For turbulence zone:
In the turbulent region, the shear stress can be expressed using empirical relations that depend on Reynolds number ($Re_x$). [10]

For turbulent flow ($Re_x > 5\ X\ 10^5$), the shear stress is approximately

$$\tau_w = 0.0225\rho U_\infty{}^2\frac{1}{Re^{\frac{1}{5}}}$$

### 2.9. coefficient skin friction ($\mathrm{C}_{fx}$)

in the context of airflow over a flat plate is a dimensionless parameter that quantifies the frictional resistance due to the shear stress at the surface of the plate caused by the viscous effects of the fluid. It is defined as;

$$C_{fx} = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}$$

where:

- $\tau_w$ =Wall share stress, which is the tangential force per unit area exerted by the fluid on the plate.
- $\rho$ =Density of the air(fluid)
- $U_\infty$ =Free stream velocity of the airflow far from the plate.

### 2.10. Shape factor

A shape factor is used in boundary layer flow to determine the nature of the flow. The higher the value of H, the stronger the adverse pressure gradient. [11]A high adverse pressure gradient can greatly reduce the Reynolds number at which transition into turbulence may occur. Conventionally, H = 2.59 (Blasius boundary layer) is typical of laminar flows, while H = 1.3 – 1.4 is typical of turbulent flows. can be calculated as:

$$H = \frac{\delta_x}{\delta_{ix}} \qquad [12]$$

## III. EXPERIMENTAL ANALYSIS OF THE BOUNDARY LAYER OF AIR FLOWING OVER A FLAT PLATE

To experimentally investigate the boundary layer characteristics of air flowing over a flat plate, including the measurement of the boundary layer thickness, velocity profile, and the transition from laminar to turbulence flow.

### 3.1. APPARATUS AND MATERIALS

1. Wind tunnel: to provide controlled airflow over the flat plate.
2. Flat plate: smooth surface of known dimensions.
3. Pitot tube or hot-wire anemometer: to measure velocity at various points in the boundary layer.
4. Manometer: to record pressure difference (if using a pitot tube).
5. Data acquisition system: to record and analyze velocity data.
6. Measuring scale: for precise positioning of the velocity measurement device.
7. Smoke generator(optional): to visualize the flow and observe the transition to turbulence.

### 3.2. PROCEDURE

1. Setting up the wind tunnel
- Place the flat plate horizontally in the test section of the wind tunnel.
- Ensure the plate is aligned with the free stream airflow to minimize disturbances.
2. measuring the free stream velocity
- Use the pitot tube or anemometer to measure the free stream velocity (U).
- Record the temperature and pressure of the air to account for changes in density.
3. Velocity profile measurement
- Position the pitot tube or anemometer at a fixed distance (x) from the leading edge of the plate.
- Measure the velocity (u) at varying heights (y) above the plate.
  ºstart close to the surface and move in small increments outward until reaching the free stream velocity.
- Repeat the measurements at several positions(x) along the plate to capture the development of the boundary.
4. Transition to turbulence
- Gradually increase the free-stream velocity to observe the transition from laminar to turbulence flow.
- Use smoke visualization to identify the critical transition point along the plate.
5. Boundary layer thickness (δ)
- For each position (x), determine the boundary layer thickness (δ) as the height where the velocity reaches 99% of the free-stream velocity.
- Record the data for the laminar and turbulence regions. [13]

Figure (2): Air Flow Bench.

## IV. RESULTS AND DISCUSSION

Experimental data table

### 4.1. BOUNDARY LAYER THICKNESS

Table (1): Boundary layer thickness with different positons.

| X(m) | $Re_x$ | $\delta_{larminar}$ | $\delta_{turbulence}$ | $\delta_{experimenta}$ |
|---|---|---|---|---|
| 0.1 | $6.77x10^3$ | 2.15 | 1.39 | 1.80 |
| 0.2 | $1.35x10^4$ | 3.04 | 2.16 | 2.50 |
| 0.3 | $2.03 x10^4$ | 3.72 | 2.77 | 3.10 |
| 0.4 | $2.71x10^4$ | 4.31 | 3.30 | 3.75 |
| 0.5 | $3.39x10^4$ | 4.82 | 3.77 | 4.30 |

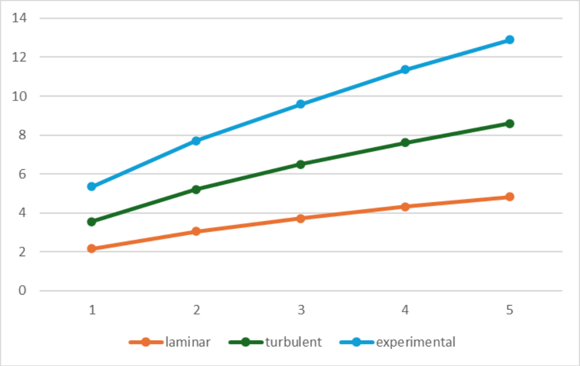

Figure (3): Boundary Layer With Local Position.

### 4.2. VELOCITY PROFILE

Table 3: The velocity profile with the boundary layer in different position.

| $\frac{y}{\delta(x}$ | $y(mm$ | $\frac{u}{U_\infty}(lami$ | $\frac{u}{U_\infty}(turbul$ | $\frac{u}{U_\infty}(experim$ |
|---|---|---|---|---|
| 0.1 | 0.48 | 0.01 | 0.19 | 0.15 |
| 0.2 | 0.96 | 0.04 | 0.34 | 0.30 |
| 0.3 | 1.45 | 0.09 | 0.45 | 0.40 |
| 0.4 | 1.93 | 0.16 | 0.54 | 0.50 |
| 0.5 | 2.41 | 0.25 | 0.62 | 0.57 |

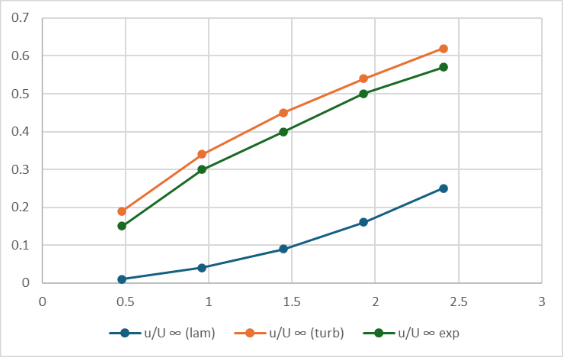

Figure (5): The velocity profile with the boundary layer.

### 4.3 LOCAL FRICTION COEFFICIENT.

Table (2) shows the relationship between the coefficient of friction and the local position, Fluids can only exert two types of forces: normal forces due to pressure and tangential forces due to shear stress. Pressure drag is the phenomenon that occurs when a body is oriented perpendicular to the direction of fluid flow. Skin friction drag is the frictional shear force exerted on a body aligned parallel to the flow, and therefore a direct result of the viscous boundary layer. Due to the greater shear stress at the wall, the skin friction drag is greater for turbulent boundary layers than for laminar ones as shown in fig(5).

Table(2): local friction coefficient with different positons.

| X (m) | $Re_x$ | $c_{flarminar}$ ( | $c_{fturbulence}$ ( | $c_{fexperimental}$ |
|---|---|---|---|---|
| 0.1 | $6.77x$ | 8.08 | 5.39 | 6.00 |
| 0.2 | $1.35x$ | 5.53 | 4.54 | 5.00 |
| 0.3 | $2.03$ $x10^4$ | 4.66 | 4.05 | 4.30 |
| 0.4 | $2.71x$ | 4.04 | 3.74 | 3.85 |
| 0.5 | $3.39x$ | 3.62 | 3.52 | 3.60 |

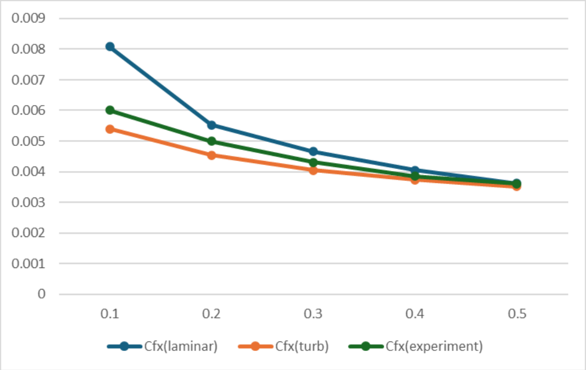

Figure (4): friction coefficient with Local Position.

### 4.4. SHAPE FACTOR

Table(2): shape factor with different positions.

| X(m) | $Re_x$ | $\delta_x$(displacement thickness) (mm) | $\delta_{ix}$(momentum thickness) (mm) | Shape factor (H) |
|---|---|---|---|---|
| 0.1 | $6.77x10$ | 1.84 | 1.42 | 1.295775 |
| 0.2 | $1.35x10$ | 2.57 | 2.03 | 1.266010 |

| | | | | |
|---|---|---|---|---|
| 0.3 | $2.03x10^4$ | 3.23 | 2.56 | 1.261719 |
| 0.4 | $2.71x10$ | 3.83 | 3.02 | 1.268212 |
| 0.5 | $3.39x10$ | 4.31 | 3.47 | 1.242075 |

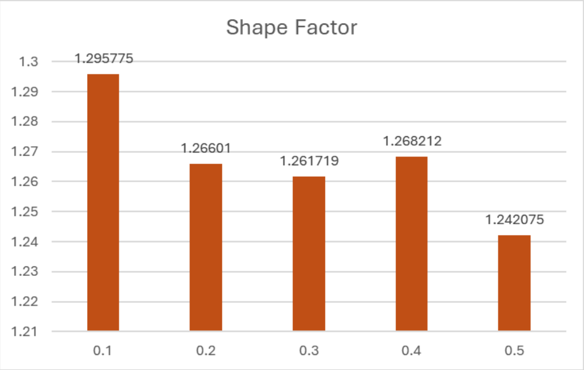

Figure (6): shape factor with local position.

## DISCUSSION

### Boundary Layer Thickness

The boundary layer thickness results presented in Table 1 and Figure 3 show that the boundary layer thickness increases with distance from the leading edge of the flat plate. The experimental results are in good agreement with the theoretical laminar and turbulent boundary layer thicknesses.

### Velocity Profile

The velocity profile results presented in Table 3 and Figure 5 show that the experimental velocity profiles exhibit characteristics of both laminar and turbulent flows. The velocity profiles are compared with the theoretical laminar and turbulent velocity profiles, and the results show that the experimental velocity profiles are in good agreement with the theoretical profiles.

### Local Friction Coefficient

The local friction coefficient results presented in Table 2 and Figure 4 show that the friction coefficient decreases with increasing distance from the leading edge of the flat plate. The experimental results are in good agreement with the theoretical laminar and turbulent friction coefficients.

### Shape Factor

The shape factor results presented in Table 2 and Figure 6 show that the shape factor remains relatively constant over the length of the flat plate. The experimental results are in good agreement with the theoretical shape factor values.

### Transition from Laminar to Turbulent Flow

The results show that the transition from laminar to turbulent flow occurs over a range of Reynolds numbers. The experimental velocity profiles and friction coefficient values exhibit characteristics of both laminar and turbulent flows, indicating a transition region.

### Implications and Applications

The findings of this study have significant implications for the design of aerodynamic systems, such as aircraft wings and wind turbines. The results provide valuable insights into the boundary layer characteristics and the transition from laminar to turbulent flow.

## CONCLUSION

In conclusion, comparisons made between experimental results and theoretical data allow the determination to be made that the boundary layer over the plate was a laminar boundary layer, and the boundary layer over plate was turbulent.

1. The differences between these two types of boundary layers were clearly demonstrated.
2. The results showed that the coefficient of skin friction decreases with the increase in the velocity of the liquid.
3. The error between theoretical and experimental for friction coefficient is 6.67%.
4. increase in the boundary layer thickness causes an increase in Reynold number.
5. The graph indicates that the skin friction coefficient increases along the length of the surface due to the development of boundary layer.
6. The skin friction coefficient for the laminar is higher than the turbulence and experimental.
7. The skin friction of the experimental, lies between the laminar and the turbulent. Indicating transitional behaviors.
8. For lower Reynold number value, the skin friction coefficient for laminar dominates suggesting laminar flow regime.

9. It indicates that in real world scenario, skin friction coefficient for experimental does not adhere to either laminar or turbulent model.
10. A plot of the skin friction coefficient to distance(x) or Reynolds number indicates a decrease trend in the curve.
11. The data shows a distinct characteristic of laminar and turbulent boundary layer.
12. Experimental results aligns mostly with turbulent flow possibly indicating the dominance of turbulence in practical scenarios.
13. The gradual transition of velocity from zero at the wall to $u_\infty$ illustrates the development of the boundary layer.
14. For flat plate, the shear factor varies.

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