Introduction of a New Parameter and Study of Coefficient Inequality in Class of Starlike Functions

GURMEET SINGH[1], CHATINDER KAUR[2]
[1] *Department of Mathematics, GSSDGS Khalsa College, Patiala*
[2] *Research fellow, Department of Mathematics, Punjabi University, Patiala.*

Abstract- In the present paper we introduce a new class of starlike functions and find the Fekete – Szegö Inequality by using the concept of subordination for the analytic function

$$f(z) = z + a_2 z^2 + a_3 z^3 + \cdots, \qquad |z| < 1$$

in the open unit disc. $U = z : z \in C; |z| < 1$

2000 Mathematical Subject Classification: 30C45, 30C50.

Indexed Terms- Analytic Function, Bounded function, Fekete – Szegö Inequality, Starlike Function, Subordination, Univalent Function.

I. INTRODUCTION

For the study of this class ofstarlikefunctions ,let us define the function f of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k$$

This is analytic in the unit disc $E = \{z; |z| < 1\}$ and denote the class of such functions by A.

let S denote the class of functions in A that are univalent in$U$.

In 1916, for the functions $f(z) \in S$, Bieber Bach [3] proved the result$|a_2| \leq 2$. In 1917 and 1923, for the same functions, Lowner ([8], [9]) proved that$|a_3| \leq 3$. With these results$|a_2| \leq 2$ and$|a_3| \leq 3$, for the class S it was very easy to draw out the relation between a_3 and a_2^2 .

Fekete and Szegö find the relationship between the bonds of second and third coefficients for the function $f(z)$and study the estimation of the upper bound of $|a_3 - \mu a_2^2|$. The well-known result by Feketeand Szegö[7] states that:

If $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ is analytic and univalent in E then for real perameter μ , we have

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3 - 4\mu & if \quad \mu \leq 0 \\ 1 + \exp\left(\frac{-2\mu}{1-\mu}\right) & if \quad 0 \leq \mu \leq 1 \\ 4\mu - 3 & if \quad \mu \geq 1 \end{cases}$$

The result is sharp, in other words for each μ there exists a function for which equality hold and that functions are called extremals.

This inequality is very much helpful in determining estimates of higher coefficients for some subclasses S (See Chhichra [5], Babalola [2]).

Starlikefunction:A function $f \in A$ is called Starlike function in the unit disc E if it is univalent in E and maps $|z| < 1$ conformally on to Starlikedomain.The subclass which contains all univalent Starlike functions, denoted by S^* was firstly studied by Alex1ander [1]. Duren [6] introduced the necessary and sufficient condition for a function $f \in A$ is to be in S^* is that

$$\mathrm{Re}\left(\frac{zf'(z)}{f(z)}\right) > 0, z \in E$$

Analytic bounded functions: Class of analytic bounded function is of the form

$$w(z) = \sum_{n=1}^{\infty} c_n z^n, w(0) = 0, |w(z)| \leq 1.$$

It is known that $\left|c_1\right| \le 1, \left|c_2\right| \le 1 - \left|c_1\right|^2$.

Definition:For the function $f(z)$ the class of Starlike functions is defined as

$$SL(pz) = \left\{ f \in S, \operatorname{Re}\left[\frac{1}{2}\left(\frac{zf'(pz)}{f(z)} + \frac{pzf'(z)}{f(pz)}\right)\right] > 0, \forall z \in E \right\}$$

Theorem 1: Let the analytic function $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ belongs the class $SL(pz)$ of starlikefunctions, then

$$\left|a_3 - \mu a_2^2\right| \le \begin{cases} \dfrac{4(p^2-p+1)+4(p^2+p)}{(3p^2-p^2-p+3)(p^2-p+1)^2} - \dfrac{4\mu}{(p^2-p+1)^2} & ; \mu \le \dfrac{(p^2+p)}{(3p^3-p^2-p+3)} \\ \dfrac{4}{(3p^2-p^2-p+3)} & ; \dfrac{(p^2+p)}{(3p^3-p^2-p+3)} < \mu < \dfrac{2(p^2-p+1)^2+(p^2+p)}{(3p^3-p^2-p+3)} \\ \dfrac{4\mu}{(p^2-p+1)^2} - \dfrac{4(p^2-p+1)+4(p^2+p)}{(3p^2-p^2-p+3)(p^2-p+1)^2} & ; \mu \ge \dfrac{2(p^2-p+1)^2+(p^2+p)}{(3p^3-p^2-p+3)} \end{cases}$$

The result is sharp.

Proof: By the definition of $SL(p,z)$, we have

$$\frac{1}{2}\left(\frac{zf'(pz)}{f(z)} + \frac{pzf'(z)}{f(pz)}\right) \prec \frac{1+w(z)}{1-w(z)}$$

By expending the series

$$\frac{1}{2}\left((1+p)+(p^2-p+1)2a_2z+\left((3p^3-p^2-p+3)a_3-(p^2+p)a_2^2\right)z^2+\cdots\right)=1+2c_1z+\left(2c_2+2c_1^2\right)z^2+\cdots \quad \ldots\ldots(1.1)$$

Comparing coefficients of (1.1)

$$a_2 = \frac{2c_1}{p^2-p+1} ,$$

$$a_3 = \frac{4}{(3p^3-p^2-p+3)}c_2 + \frac{4\left((p^2-p+1)^2+(p^2+p)\right)}{(3p^3-p^2-p+3)(p^2-p+1)^2}c_1^2 \quad \ldots(1.2)$$

$$\left|a_3-\mu a_2^2\right| = \frac{4}{(3p^3-p^2-p+3)}\left|c_2\right| + \left\{\left|\frac{4\left((p^2-p+1)^2+(p^2+p)\right)}{(3p^3-p^2-p+3)(p^2-p+1)^2} - \frac{4\mu}{(p^2-p+1)^2}\right|\right\}\left|c_1^2\right|$$

$$\left|a_3-\mu a_2^2\right| = \frac{4}{(3p^3-p^2-p+3)} + \left\{\left|\frac{4\left((p^2-p+1)^2+(p^2+p)\right)}{(3p^3-p^2-p+3)(p^2-p+1)^2} - \frac{4\mu}{(p^2-p+1)^2}\right| - \frac{4}{(3p^3-p^2-p+3)}\right\}\left|c_1^2\right| \quad \ldots(1.3)$$

Case 1: when $\mu \le \dfrac{(p^2-p+1)^2+(p^2+p)}{3p^3-p^2-p+3}$

Inequality (1.3) can be rewritten as

$$\left|a_3-\mu a_2^2\right| = \frac{4}{(3p^3-p^2-p+3)} + \left\{\frac{4(p^2+p)}{(3p^3-p^2-p+3)(p^2-p+1)^2} - \frac{4\mu}{(p^2-p+1)^2}\right\}\left|c_1^2\right| \quad \ldots(1.4)$$

Sub case 1(a): When $\mu \le \dfrac{(p^2+p)}{3p^3-p^2-p+3}$

Then equation (1.4) can be rewritten as

$$\left|a_3-\mu a_2^2\right| \le \frac{4\left((p^2+p)+(p^2-p+1)^2\right)}{(3p^3-p^2-p+3)(p^2-p+1)^2} - \frac{4\mu}{(p^2-p+1)^2} \quad \ldots(1.5)$$

Sub case 1(b): When $\dfrac{(p^2-p+1)^2+(p^2+p)}{3p^3-p^2-p+3} > \mu > \dfrac{(p^2+p)}{3p^3-p^2-p+3}$

then the equation (1.4) becomes

$$\left|a_3-\mu a_2^2\right| \le \frac{4}{3p^3-p^2-p+3} \quad \ldots(1.6)$$

Case2: When $\mu \ge \dfrac{(p^2-p+1)^2+(p^2+p)}{3p^3-p^2-p+3}$

$$\left|a_3-\mu a_2^2\right| = \frac{4}{(3p^3-p^2-p+3)} + \left\{\frac{4\mu}{(p^2-p+1)^2} - \frac{4\left((p^2-p+1)^2+(p^2+p)\right)}{(3p^3-p^2-p+3)(p^2-p+1)^2} - \frac{4}{(3p^3-p^2-p+3)}\right\}\left|c_1^2\right| \quad \ldots(1.7)$$

Sub case 2(a): When $\mu \ge \dfrac{2(p^2-p+1)+(p^2+p)}{3p^3-p^2-p+3}$

Then the equation (1.7) becomes

$$\left|a_3-\mu a_2^2\right| \le \frac{4\mu}{(p^2-p+1)^2} - \frac{4\left((p^2+p)+(p^2-p+1)^2\right)}{(3p^3-p^2-p+3)(p^2-p+1)^2} \quad \ldots(1.8)$$

Sub case 2(b): When $\dfrac{2(p^2-p+1)^2+(p^2+p)}{3p^3-p^2-p+3} \ge \mu \ge \dfrac{(p^2-p+1)+(p^2+p)}{3p^3-p^2-p+3}$

Then the equation (1.8) becomes

$$\left|a_3-\mu a_2^2\right| \le \frac{4}{3p^3-p^2-p+3} \quad \ldots(1.9)$$

Combining the equations (1.5), (1.6), (1.8) and (1.9). We get the required coefficient inequality for $SL(pz)$

.

The Extremal function for first and third inequality is

$$f_1(z) = z\left\{1+\frac{\left(2(3p^3-3p^2-3p+3)^2-4(p^2-p+1)^2\right)z}{(p^2-p+1)(3p^3-p^2-p+3)}\right\}^{\frac{3p^3-p^2-p+3}{(3p^3-3p^2-3p+3)^2-2(p^2-p+1)^2}}$$

Extremal function for second inequality is

$$f_2(z) = z\left\{1+\frac{1}{3p^3-p^2-p+1}z^2\right\}^4$$

Corollary 1.1: Putting $p=1$ in $SL(pz)$ we get

$$\left|a_3-\mu a_2^2\right|\le\begin{cases}3-4\mu & if \quad \mu\le\frac{1}{2};\\ 1 & if \quad \frac{1}{2}<\mu<1;\\ 4\mu-3 & if \quad \mu\ge 1\end{cases}$$

Which is the result obtained by [7].

Subclass $SL(\delta, pz)$ of class $SL(pz)$

Now we introducethe subclass $SL(\delta, pz)$ of class $SL(pz)$, which satisfies the condition

$$\mathrm{Re}\left(\frac{1}{2}\left(\frac{zf'(pz)}{f(z)}+\frac{pzf'(z)}{f(pz)}\right)\right)>0 \quad z\in E$$

$$i.e. \quad \frac{1}{2}\left(\frac{zf'(pz)}{f(z)}+\frac{pzf'(z)}{f(pz)}\right)\prec\left(\frac{1+w(z)}{1-w(z)}\right)^{\delta}$$

Theorem 2: Let the analytic function $f(z)=z+\sum_{n=2}^{\infty}a_nz^n$ belongs to the set of the class of starlikefunctions $SL(\delta, pz)$, then

$$\left|a_3-\mu a_2^2\right|\le\begin{cases}\frac{4\delta^2(p^2-p+1)+4\delta^2(p^2+p)}{(3p^2-p^2-p+3)(p^2-p+1)^2}-\frac{4\delta^2\mu}{(p^2-p+1)^2} & ;\mu\le\frac{(\delta-1)(p^2-p+1)^2+\delta(p^2+p)}{\delta(3p^3-p^2-p+3)}\\ \frac{4\delta^2}{(3p^2-p^2-p+3)} & ;\frac{(\delta-1)(p^2-p+1)^2+\delta(p^2+p)}{\delta(3p^3-p^2-p+3)}<\mu<\frac{(\delta+1)(p^2-p+1)^2+\delta(p^2+p)}{\delta(3p^3-p^2-p+3)}\\ \frac{4\delta^2\mu}{(p^2-p+1)^2}-\frac{4\delta^2(p^2-p+1)+4\delta^2(p^2+p)}{(3p^2-p^2-p+3)(p^2-p+1)^2} & ;\mu\ge\frac{(\delta+1)(p^2-p+1)^2+\delta(p^2+p)}{\delta(3p^3-p^2-p+3)}\end{cases}$$

The result is sharp.

Proof: By the definition of $SL(\delta, pz)$, we have

$$i.e. \quad \frac{1}{2}\left(\frac{zf'(pz)}{f(z)}+\frac{pzf'(z)}{f(pz)}\right)\prec\left(\frac{1+w(z)}{1-w(z)}\right)^{\delta}$$

By expending the series

$$\frac{1}{2}\left(((1+p)+(p^2-p+1))2a_2z+\left((3p^3-p^2-p+3)a_3-(p^2+p)a_2^2\right)z^2+\cdots\right)=1+2\delta c_1z+\left(2\delta c_2+2\delta^2c_1^2\right)z^2+\cdots \quad \ldots\ldots(2.1)$$

Comparing coefficients of (2.1)

$$a_2=\frac{2\delta c_1}{p^2-p+1} \quad ,$$

$$a_3=\frac{4\delta}{(3p^3-p^2-p+3)}c_2+\frac{4\delta^2\left((p^2-p+1)^2+(p^2+p)\right)}{(3p^3-p^2-p+3)(p^2-p+1)^2}c_1^2$$

$$\left|a_3-\mu a_2^2\right|=\frac{4\delta}{(3p^3-p^2-p+3)}|c_2|+\left\{\left|\frac{4\delta^2\left((p^2-p+1)^2+(p^2+p)\right)}{(3p^3-p^2-p+3)(p^2-p+1)^2}-\frac{4\delta^2\mu}{(p^2-p+1)^2}\right|\right\}\left|c_1^2\right|$$

$$\left|a_3-\mu a_2^2\right|=\frac{4\delta}{(3p^3-p^2-p+3)}+\left\{\left|\frac{4\delta^2\left((p^2-p+1)^2+(p^2+p)\right)}{(3p^3-p^2-p+3)(p^2-p+1)^2}-\frac{4\delta\mu}{(p^2-p+1)^2}\right|-\frac{4\delta}{(3p^3-p^2-p+3)}\right\}\left|c_1^2\right| \quad \ldots(2.2)$$

Case 1: when $\mu\le\frac{(p^2-p+1)^2+(p^2+p)}{3p^3-p^2-p+3}$

Inequality (2.2) can be rewritten as

$$\left|a_3-\mu a_2^2\right|=\frac{4\delta}{(3p^3-p^2-p+3)}+\left\{\frac{4\delta^2\left((p^2-p+1)^2+(p^2+p)\right)-4\delta(p^2-p+1)^2}{(3p^3-p^2-p+3)(p^2-p+1)^2}-\frac{4\delta\mu}{(p^2-p+1)^2}\right\}\left|c_1^2\right| \ldots(2.3)$$

Sub case 1(a): When $\mu\le\frac{(\delta-1)(p^2-p+1)^2+\delta(p^2+p)}{(3p^3-p^2-p+3)\delta}$

Then equation (2.3) can be rewritten as

$$\left|a_3-\mu a_2^2\right|\le\frac{4\delta^2\left((p^2+p)+(p^2-p+1)^2\right)}{(3p^3-p^2-p+3)(p^2-p+1)^2}-\frac{4\delta^2\mu}{(p^2-p+1)^2} \quad \ldots(2.4)$$

Sub case 1(b): When $\frac{(p^2-p+1)^2+(p^2+p)}{3p^3-p^2-p+3}>\mu>\frac{(\delta-1)(p^2-p+1)^2+\delta(p^2+p)}{(3p^3-p^2-p+3)\delta}$

then the equation (2.3) becomes

$$\left|a_3-\mu a_2^2\right|\le\frac{4\delta}{3p^3-p^2-p+3} \quad \ldots(2.5)$$

Case2: When $\mu\ge\frac{(p^2-p+1)^2+(p^2+p)}{3p^3-p^2-p+3}$

$$\left|a_3-\mu a_2^2\right|=\frac{4\delta}{(3p^3-p^2-p+3)}+\left\{\frac{4\delta^2\mu}{(p^2-p+1)^2}-\frac{4\delta^2\left((p^2-p+1)^2+(p^2+p)\right)+4\delta(p^2-p+1)^2}{(3p^3-p^2-p+3)(p^2-p+1)^2}\right\}\left|c_1^2\right| \ldots(2.6)$$

Sub case 2(a): When $\mu\ge\frac{(\delta+1)(p^2-p+1)^2+\delta(p^2+p)}{\delta(3p^3-p^2-p+3)}$

Then the equation (2.6) becomes

$$\left|a_3-\mu a_2^2\right|\le\frac{4\delta^2\mu}{(p^2-p+1)^2}-\frac{4\delta^2\left((p^2+p)+(p^2-p+1)^2\right)}{(3p^3-p^2-p+3)(p^2-p+1)^2} \quad \ldots(2.7)$$

Sub case 2(b): When $\frac{(\delta+1)(p^2-p+1)^2+\delta(p^2+p)}{(3p^3-p^2-p+3)\delta}\ge\mu\ge\frac{(p^2-p+1)+(p^2+p)}{3p^3-p^2-p+3}$

Then the equation (2.6) becomes

$$\left|a_3-\mu a_2^2\right|\le\frac{4\delta}{3p^3-p^2-p+3} \quad \ldots(2.8)$$

Combining the equations (2.4), (2.5), (2.7) and (2.8).

We get the required coefficient inequality for $SL(\delta, pz)$

The Extremal function for first and third inequality is

$$f_1(z) = z\left\{1 + \frac{\left(2\delta(3p^3 - 3p^2 - 3p + 3)^2 - 4\delta(p^2 - p + 1)^2\right)z}{(p^2 - p + 1)(3p^3 - p^2 - p + 3)}\right\}^{\frac{3p^3 - p^2 - p + 3}{(3p^3 - 3p^2 - 3p + 3)^2 - 2(p^2 - p + 1)^2}}$$

Extremal function for second inequality is

$$f_2(z) = z\left\{1 + \frac{\delta}{3p^3 - p^2 - p + 1} z^2\right\}^4$$

Corollary 2.1: Putting $\delta = 1$ in $SL(\delta, pz)$ we get

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{4(p^2 - p + 1) + 4(p^2 + p)}{(3p^2 - p^2 - p + 3)(p^2 - p + 1)^2} - \frac{4\mu}{(p^2 - p + 1)^2} & ; \mu \leq \frac{(p^2 + p)}{(3p^3 - p^2 - p + 3)} \\ \frac{4}{(3p^2 - p^2 - p + 3)} & ; \frac{(p^2 + p)}{(3p^3 - p^2 - p + 3)} < \mu < \frac{2(p^2 - p + 1)^2 + (p^2 + p)}{(3p^3 - p^2 - p + 3)} \\ \frac{4\mu}{(p^2 - p + 1)^2} - \frac{4(p^2 - p + 1) + 4(p^2 + p)}{(3p^2 - p^2 - p + 3)(p^2 - p + 1)^2} & ; \mu \geq \frac{2(p^2 - p + 1)^2 + (p^2 + p)}{(3p^3 - p^2 - p + 3)} \end{cases}$$

We get $SL(\delta, pz) = SL(pz)$ for $\delta = 1$.

Corollary 2.2: Putting $p = 1$ and $\delta = 1$ in the theorem 2 we get

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3 - 4\mu & \text{if} \quad \mu \leq \frac{1}{2}; \\ 1 & \text{if} \quad \frac{1}{2} < \mu < 1; \\ 4\mu - 3 & \text{if} \quad \mu \geq 1 \end{cases}$$

Which is the result obtained by [7].

REFERENCES

[1] Alexander J. W., *Function which map the interior of unit circle upon simple regions*, Ann. Math., Vol. 17, pp. 12-22.

[2] Bieberbach, L., (1916), *UberKoeffizientenderjenigenPotenzreihen*, welcheeineschlichte des Einheitskreisesvermitteln, s.b. prenss. Akad.Wiss., Vol. 38, pp 940-955.

[3] BabalolaK. O. and OpoolaT. O. (2006), *On the coefficients of certain analytic and univalent functions*, Adv. Ineq. Ser. (Edited by S.S. Dragomir and A. Sofo) Nova Science Publishers., pp5-17.

[4] Caratheodory C. (1911), *Uber den VariabilitatsbEereich der Fourier schenKonstanten non positive harmonischenFunktionen*, Rend. Cire.Mat.Palermo, Vol. 32, 193-213.

[5] Chhichra P.N.(1977), New subclasses of the class of close-to-convex function, American Mathematical Society, Vol 62, 37-43.

[6] Duren P.L. (1977), *Coefficient of univalent functions*, Bulletin of American Mathematical Society, Vol. 83, pp.891-911.

[7] Fekete, M. and Szeg¨ o, G. (1933), *EineBemerkunguberungeradeschlichtefunktionen*, J. London Math. Soc., Vol. 8, pp 85-89.

[8] Loewner, K.(1917), *Untersuchungenuber die VerzerrungbeiKonformenAbbildungen des Einheitskreises |z| < 1, die durchFunktionenmitnichtverschwindenderAbleitunggeliefert warden* , Ber. Verh. Sachs.Ges.wiss. Leipzig, Vol. 69, pp 89-106.

[9] Loewner, K., (1923), *UntersuchungenuberschlichtekonformeAbbildungen des Einheitskreises I*, Math. Ann. Vol. 89, pp 103-121.

[10] Kaur, C. and Singh, G., Approach To Coefficient Inequality For A New Subclass Of Starlike Functions With Extremals, *International Journal Of Research In Advent Technology*, 5(2017)

[11] Kaur, C. and Singh, G., Coefficient Problem For A New Subclass Of Analytic Functions Using Subordination, *International Journal Of Research In Advent Technology*, 5(2017)

[12] Kaur. G, Singh. G, Arif. M, Chinram. R, Iqbal. J, A study of third and fourth Hankel determinant problem for a particular class of bounded turning functions, *Mathematical Problems in Engineering*, 22, 511-526, 2021

[13] Kaur N., Fekete Szego Inequality Alongwith Their Extremal Functions Making Results Sharp for Certain Subclasses Of Analytic Functions, *Webology*, 18(4), 2021, 3075-3083

[14] Keogh, F.R., Merkes, E.P., A coefficient inequality for certain classes of analytic functions, *Proc. Of Amer. Math. Soc.*, 20, 8-12, 1989.

[15] Koebe, P., *Ueber die* Uniformisierung beliebiger analytischer Kurven." Nachrichten *von der* Gesellschaft *der* Wissenschaften zu Göttingen, (1907), 633-669.

[16] Ma, W. and Minda, D. unified treatment of some special classes of univalent functions, *In Proceedings of the Conference on Complex Analysis* , Int. Press Tianjin (1994), 157-169.

[17] Miller, S.S., Mocanu, P.T. And Reade, M.O., All convex functions are univalent and starlike, *Proc. of Amer. Math. Soc.*, 37 (1973), 553-554.

[18] Nehari, Z. (1952), *Conformal Mappings*, McGraw- Hill, New York.

[19] Nevanlinna, R., Über die konforme Abbildung von Sterngebieten. Översikt av Finska Vetenskaps-Soc. Förh. 63(A), Nr. 6, 1–21, 1920

[20] Pederson, R., A proof for the Bieberbach conjecture for the sixth coefficient,*Arch. Rational Mech. Anal*., 31 (1968-69), 331-351.

[21] Pederson, R. and Schiffer, M., A proof for the Bieberbach conjecture for the fifth coefficient, *Arch. Rational Mech. Anal.*, 45 (1972), 161-193.

[22] Rani, M., Singh, G., Some Classes Of Schwarzian Functions And Its Coefficient Inequality That Is Sharp, *Turk. Jour. Of Computer and Mathematics Education*, 11 (2020), 1366-1372.

[23] Rathore, G. S., Singh, G. and Kumawat, L. et.al., Some Subclaases Of A New Class Of Analytic Functions under Fekete-Szego Inequality, *Int. J. of Res. In Adv. Technology*, 7(2019)

[24] Rathore. G. S., Singh, G., Fekete – Szego Inequality for certain subclasses of analytic functions , *Journal Of Chemical , Biological And Physical Sciences*, 5(2015) ,

[25] Singh. G, Fekete – Szego Inequality for a new class and its certain subclasses of analytic functions , *General Mathematical Notes*, 21 (2014),

[26] Singh. G, Fekete – Szego Inequality for a new class of analytic functions and its subclass, *Mathematical Sciences: International Research Journal*, 3 (2014),

[27] Singh. G., Construction of Coefficient Inequality For a new Subclass of Class of Starlike Analytic Functions, *Russian Journal of Mathematical Research Series*, 1 (2015), 9-13.

[28] Singh, G., Introduction of a new class of analytic functions with its Fekete–Szegö Inequality, *International Journal of Mathematical Archive*, 5 (2014), 30-35.

[29] Singh, G, An Inequality of second and third Coefficients for a Subclass of Starlike Functions Constructed Using nth Derivative, *Kaav Int.J. Of Sci. Eng. And Tech.*, 4 (2017), 206-210.

[30] Singh, G, Fekete–Szego Inequality for asymptotic subclasses of family of analytic functions, *Stochastic Modelling And Applications*, 26 (2022),

[31] Singh, G, Coefficient Inequality for Close to Starlike Functions Constructed Using Inverse Starlike Classes, *Kaav Int. J. Of Sci. Eng. And Tech.*, 4 (2017), 177-182.

[32] Singh, G, Coeff. Inequality for a subclass of Starlike functions that is constructed using nth derivative of the functions in the class, *Kaav Int. J. Of Sci. Eng. And Tech.*, 4 (2017), 199-202.

[33] Singh G., Fekete Szego Inequality For A Complicated Class Of Analytic Functions Approaching To A Class In The Limit Form And Other Class Directly, *IJRES*, 10 (9), 619-624, 2022

[34] Singh, G., Fekete–Szegö Inequality for functions approaching to a class in the limit form and another class directly, *Journal Of Information And Computational Sciences*, 12 (4), 2022, 181-186

[35] Singh G., Bansal K., Kaur H., Fekete Szego Coefficient Inequality for a Subclass of Analytic Functions, *International Journal of Advances in Engineering and Management (IJAEM)*, 4 (10), 2022, 1295-1298

[36] Singh, G., Garg, J., Coefficient Inequality For A New Subclass Of Analytic Functions, *Mathematical Sciences: International Research Journal*, 4(2015)

[37] Singh G, Sharma N., Two new subclasses of already defined class of Analytic functions and establishment of their coefficient inequality, NeuroQuantology, 20 (10), 4967-4976, 2022

[38] Singh, G, Singh, Gagan, Fekete–Szegö Inequality For Subclasses Of A New Class Of Analytic Functions, *Proceedings Of The World Congress On Engineering* , (2014) , .

[39] Singh G, Houque M. A., A specially designed class of univalent functions and establishment of its coefficient inequality, *NeuroQuantology*, 20 (10), 2855-2859, 2022

[40] Singh G, Kaur C, Analytic functions subordinate to leaf-like domain, *Advances in Mechanics*, 10 (1), 1444-1448, 2022

[41] Singh, G, Sarao, M. S., and Mehrok, B. S., Fekete – Szegö Inequality For A New Class Of Analytic Functions , *Conference Of Information And Mathematical Sciences* , (2013).

[42] Singh. G, Singh. Gagan, Sarao. M. S., Fekete – Szegö Inequality for a New Class of Convex Starlike Analytic Functions, *Conf. Of Information and Mathematical Sciences*, (2013).

[43] Singh, G., Kaur, G., Coefficient Inequality for a Subclass of Starlike Function generated by symmetric points, *Ganita*, 70 (2020), 17-24.

[44] Singh ,G., Kaur, G., Coefficient Inequality For A New Subclass Of Starlike Functions, *International Journal Of Research In Advent Technology,* 5, (2017) ,

[45] Singh, G., Kaur, G., Fekete-Szegö Inequality For A New Subclass Of Starlike Functions, *International Journal Of Research In Advent Technology,* 5, (2017) ,

[46] Singh, G., Kaur, G., Fekete-Szegö Inequality For Subclass Of Analytic Function Based On Generalized Derivative, *Aryabhatta Journal Of Mathematics And Informatics,* 9 (2017),

[47] Singh, G., Kaur, G., Coefficient Inequality For a subclass of analytic function using subordination method with extremal function, *Int. J. Of Advance Res. In Sci & Engg* , 7 (2018)

[48] Singh, G., Kaur, G., Arif, M., Chinram R, Iqbal J, A study of third and fourth Hankel determinant problem for a particular class of bounded turning functions, *Mathematical Problems in Engineering*, 2021

[49] Singh, G. and Kaur, G., 4th Hankel determinant for α bounded turning function, *Advances in Mathematics: Scientific Journal*, 9 (12), 10563-10567

[50] Singh, G., Kaur, G., Fourth Hankel determinant for a new subclass of bounded turning functions, *Jnanabha*, 52 (1), 229-233, 2022 5rf vhtfvv

[51] Singh, G., Kaur, N., Fekete-Szegö Inequality For Certain Subclasses Of Analytic Functions, *Mathematical Sciences: International Research Journal*, 4(2015)

[52] Singh G, Patil A. S., An extraordinary class of asymptotic analytical functions with coefficient inequality, *NeuroQuantology*, 20 (10), 4960-4966, 2022

[53] Singh, G, Singh, B, Fekete Szego Coefficient Inequality of Regular Functions for A Special Class, *International Journal of Research in Engineering and Science*, 10(8), 2022, 556-560

[54] Singh, G, Singh, P., Fekete – Szegö Inequality For Functions Belonging To A Certain Class Of Analytic Functions Introduced Using Linear Combination Of Variational Powers Of Starlike And Convex Functions, *Journal Of Positive School Psychology*, 6 (2022), 8387-8391.

[55] Singh, G, Singh, P., fekete-szegö inequality for class of close to inverse starlike analytic functions, *Advances in Mechanics*, 10 (1), 1375-1386

[56] Singh. G, Rani M, An advance subclass of Analytic Functions having a unique coefficient inequality, *Int. J. of Research in Engineering and Science*, 10 (8), 2022, 474-476

[57] Singh, G., Singh, G., Singh, G., A subclass of bi-univalent functions defined by generalized Sãlãgean operator related to shell-like curves connected with Fibonacci numbers, *International Journal of Mathematics and Mathematical Sciences*, 2019

[58] Singh, G., Singh, G., Singh, G., A new subclass of univalent functions, *Уфимский математический журнал*, 11(1), 132-139, 2019

[59] Singh, G., Singh, G., Singh, G., A generalized subclass of alpha convex biunivalent functions of complex order, *Jnanabha*, 50 (1), 65-71, 2020

[60] Singh, G., Singh, G., Singh, G., Upper bound on fourth Hankel determinant for certain subclass of multivalent functions, *Jnanabha*, 50 (2), 122-127, 2020

[61] Singh, G., Singh, G., Singh, G., Certain subclasses of univalent and biunivalent functions related to shell-like curves connected with

Fibonacci numbers, *General Mathematics*, 28 (1), 125-140, 2020

[62] Singh, G., Singh, G., Singh, G., Certain subclasses of Sakaguchitype bi-univalent functions, *Ganita*, 69 (2), 45-55, 2019

[63] Singh, G., Singh, G., Singh, G., Certain Subclasses of Bi-Close-to-Convex Functions Associated with Quasi-Subordination, *Abstract and Applied Analysis*, 1, 1-6, 2019

[64] Singh, G., Singh, G., Singh, G., Fourth Hankel determinant for a subclass of analytic functions defined by generalized S˘al˘agean operator, *Creat. Math. Inform.*, 31(2), 229-240, 2022

[65] Singh, G., Singh, G., Singh, G., Certain subclasses of multivalent functions defined with generalized Salagean operator and related to sigmoid function and lemniscate of Bernoulli, *J. Frac. Calc. Appl*, 13 (1), 65-81, 2022

[66] Singh, G., Singh, G., Singh, G., Estimate of Third and Fourth Hankel Determinants for Certain Subclasses of Analytic Functions., *Southeast Asian Bulletin of Mathematics*, 47 (3), 2023

[67] Srivastava H. M., G. Kaur, Singh. G, Estimates of fourth Hankel determinant for a class of analytic functions with bounded turnings involving cardioid domains, *Journal of Nonlinear and Convex Analysis*, 22 (3), 511-526, 2021