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New Integral Transform "Double Kushare Transform"

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Abstract- In this paper we are introducing double Kushare integral transform. To prove the efficiency, accuracy and capability of the double Kushare transform, we solve the boundary Value problems on Partial Differential equations.

Indexed Terms- Double Kushare transform, Integral transform,

I. INTRODUCTION

Nowadays integral transforms play very important role in mathematics. Many researchers are engaged in developing new integral transforms. Recently, S.R Kushare and D. P. Patil [1] introduce Kushare transform in September 2021.In October 2021, S.S.Khakale and D. P. Patil [2] introduce Soham transform. As researchers are going to introduce new integral transforms at the same time many researchers are interested to apply these transforms to various types of problems. In January 2022 , Rohidas S. Sanap and D.. P. Patil [3] used Kushare transform to solve the problems based on Newton's law of cooling. In April 2022 D. P. Patil et al. [4] used Kushare transform to solve the problems on growth and decay. In October 2021 D. P. Patil [5] used Sawi transform in Bessel function. D. P. Patil [6] used Sawi transform of error function for evaluating improper integral further, Lpalce and Shenu transforms are used in chemical science by D. P. Patil [7]. Dr. Patil [8] solved the wave equation by Sawi transform and its convolution theorem. Further Patil [9] also used Mahgoub transform for solving parabolic boundary value problems.

Dr. Dinkar Patil [10] obtains solution of the wave equation by using double Laplace and double Sumudu transform. Dualities between double integral transforms are derived by D. P. Patil [11]. Laplace, Elzaki, and Mahgoub transforms are used for solving system of first order and first degree differential equations by Kushare and Patil [12].Boundary value problems of the system of ordinary differentiable equations are by using Aboodh and Mahgoub transform by D. P. Patil [13]. D. P. Patil [14] study Laplace, Sumudu, Elzaki and Mahgoub transforms comparatively and apply them in Boundary value problems. Parabolic Boundary value problems are also solved by Dinkar Patil [15]. For that he used double Mahgoub transform.

Soham transform is used to obtain the solution of system of differential equations by D. P. Patil et al [16]. D. P. Patil et al also used Soham transform for solving Volterra integral equations of first kind [17]. D. P. Patil et al [18] used Anuj transform to solve Volterra integral equations of first kind.. Recently Zankar, Kandekar and D. P. Patil used general integral transform of error function for evaluating improper integrals [19]. Recently, Dinkar Patil, Prerana Thakare and Prajakta Patil [20] used double general integral transform for obtaining the solution of parabolic boundary value problems. D. P. Patil et al [21] used emad-Sara transform to obtain the solution of telegraph equation. . Shirsath, Gangurde and Patil [22] Applied Soham transform for solving the problems based on Newton's law of cooling. D. P. Patil et al [23] used the HY integral transform for handling growth and Decay problems. Komal Patil, Snehal Patil and Dinkar Patil [24] solved Newton's law of cooling by using " Emad- Falih Transform" . HY transform is used for solving problems on Newton's law of cooling by Dinkar Patil et al [25].. Elzaki et al [26] introduced double elzaki transform. Thangavellu et al [27] used double Mahgoub transform in telegraph equation. D. P. Patil et al [28] used Emad-Falih transform for general solution of telegraph equation

Paper is organised as follows: Second section is reserved for preliminaries. Double Kushare transform is introduced in third section. Applications are in fourth section.

II. PRELIMINARY

In this section we state basic concepts which are required.

2.1 Definition of KUSHARE Transform:

Kushare transform of function $f(t)$ is denoted by $kf(t) = S(v)$ and it is defined as,

K[$f(t)$] =S(v)=$v\int_0^\infty f(t)e^{-tv^\alpha}$dt, t≥0

Where α is any non-zero real numbers. The variable v in this vital change is utilized to figure the variable t the contention of the capacity v

2.2 KUSHARE Transform of some functions:

Sr. No.	Function $f(t)$	$K(f(t))$
1	1	$\frac{1}{v^{(\alpha-1)}}$
2	t^n	$\frac{\Gamma(n+1)}{v^{\alpha(n+1)-1}}$
3	e^{at}	$\frac{v}{v^\alpha - a}$
4	Sin at	$\frac{av}{v^{2\alpha}+a^2}$
5	Cos at	$\frac{v^{\alpha+1}}{v^{2\alpha}+a^2}$
6	$f'(t)$	$v^\alpha s(v) - vf(0)$

III. DOUBLE KUSHARE TRANSFORM

Definition of Double KUSHARE Transform

Double Kushare transform of a function $f(x,y)$ is defined by following equation

$$K_2[f(x,y)] = v_1 v_2 \int_0^\infty \int_0^\infty f(x,y) e^{-(v_1^\alpha x + v_2^\alpha y)}\, dx\, dy, \quad x,y \geq 0$$

Where α is any non-zero real numbers. The variable v in this vital change is utilized to figure the variable t the contention of the capacity v. This necessary change has further association with the Mahgoub, Pourreza, Elzaki changes.

Notes:

(1) If α =1 then eq. becomes

$k_2[f(x,y)]$ = S($v_{1,}v_2$) =

$v_1 v_2 \int_0^\infty \int_0^\infty f(x,y)\, e^{-(v_1 x + v_2 y)}\, dx\, dy,$ x,y≥ 0,

This integral transform is called “Double Mahgoub Transform”.[29]

(2) If α = -1 then eq. becomes

$k_2[f(x,y)]$ = S($v_{1,}v_2$) =

$v_1 v_2 \int_0^\infty \int_0^\infty f(x,y)\, e^{-(\frac{x}{v_1}+\frac{y}{v_2})}\, dx\, dy,$ x,y≥ 0,

This integral transform is called “Double Elzaki Transform”.[28]

3.1. Properties of Double new KUSHARE integral transform:

a) Linearity property:

$K_2\{af(x,y)+bg(x,y)\}$ = a$K_2\{f(x,y)\}$+b$K_2\{g(x,y)\}$

Proof:

L.H.S. = K_2\{a$f(x,y) + bg(x,y)$)

$$= v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)}(af(x,y) + bg(x,y))\, dx\, dy$$

=

$v_1 v_2 (\int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)}af(x,y)\, dx\, dy$+ $\int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)} bg(x,y)\, dx\, dy)$

$= a(v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)} f(x,y)\, dx\, dy)$ + b $(v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)} g(x,y)\, dx\, dy)$

= a$K_2\{f(x,y)\}$+b$K_2\{g(x,y)\}$

= R.H.S.

b) Shifting property:

If K($f(x,y)$) = K then $K_2(e^{-(\mathrm{ax+by})}f(x,y))$= K (a, b)

That is $K_2(e^{-(\mathrm{ax+by})}f(x,y))$ = $v_1 v_2 \int_0^\infty \int_0^\infty e^{-[(v_1^\alpha + a)x + [(v_2^\alpha + b)y]} f(x,y)\, dx\, dy$

Proof:

L.H.S. = $K_2(e^{-(\mathrm{ax+by})}f(x,y))$

$=v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)} e^{-(ax+by)} f(x,y)\, dx\, dy$

$=v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)+(ax+by)} f(x,y)\, dx\, dy$

$=v_1 v_2 \int_0^\infty \int_0^\infty e^{-[(v_1^\alpha + a)x + (v_2^\alpha + b)y]} f(x,y)\, dx\, dy$

L.H.S. = R.H.S.

c) Change of scale property:

If $K_2\{f(x,y)\}$= $K_2\,(a,b)$ then $K_2\{f(ax,by)\}$= $\frac{1}{ab}K_2$ (a, b)

L.H.S.=$K_2\{f(ax,by)\}$

Substituting ax=u_1 and by=u_2 in the above equation
Hence as x→0, $u_1 \to \infty$ and y→ $\infty, u_2 \to \infty$
a$dx = du_1 \Rightarrow dx = \frac{du_1}{a}$
and b$dy = du_2 \Rightarrow dy = \frac{du_2}{b}$

$\therefore L.H.S.$

$$= v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha \frac{u_1}{a} + v_2^\alpha \frac{u_2}{b})} f(u_1 u_2) \frac{du_1}{a} \frac{du_2}{b}$$

$$= \frac{1}{ab}[v_1 v_2 \int_0^\infty \int_0^\infty e^{-\left(\left(\frac{v_1^\alpha}{a}\right)u_1 + \left(\frac{v_2^\alpha}{b}\right)u_2\right)} f(u_1 u_2)\, du_1\ du_2$$

$$= \frac{1}{ab}[v_1 v_2 \int_0^\infty \int_0^\infty e^{-(r_1^\alpha u_1 + r_2^\alpha u_2)} f(u_1 u_2)\, du_1\ du_2$$

Where $r_1^\alpha = \frac{v_1^\alpha}{a}$ *and* $r_2^\alpha = \frac{v_2^\alpha}{b}$

$=\frac{1}{ab} K_2$ (a, b)

3.2. Formulae for some elementary function:

In this section we shall derive some formulae for some elementary function by using double new KUSHARE integral transform

Formula 1)
If $f(x,y)$=1 for x>0 and y>0
$K_2\{1\} = v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)}\, dx\, dy$

$=v_1 v_2 (\int_0^\infty \int_0^\infty e^{-(v_1^\alpha x)} e^{-(v_2^\alpha y)}\, dx\, dy\ )$

$=(v_1 \int_0^\infty e^{-(v_1^\alpha x)}\, dx)(\ v_2 e^{-(v_2^\alpha y)}\, dy)$ $=(\frac{v_1}{v_1^\alpha})(\frac{v_2}{v_2^\alpha})$ $=\frac{v_1 v_2}{v_1^\alpha v_2^\alpha}$

$$\therefore K_2(1) = \frac{v_1 v_2}{v_1^\alpha v_2^\alpha}$$

Formula 2)

If $f(x,y)$=exp$(ax+by)$

$K_2(\exp(ax+by))$

$$= v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)}\, e^{(ax+by)}\, dx\, dy$$

$$= v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha - ax - by)}\, dx\, dy$$

$$= v_1 v_2 \int_0^\infty \int_0^\infty e^{-[(v_1^\alpha - a)x + (v_2^\alpha y - b)y]}\, e^{(ax+by)}\, dx\, dy$$

$$= v_1 v_2 \int_0^\infty e^{-((v_1^\alpha - a)x)} \int_0^\infty e^{-(v_2^\alpha - b)y}\, dx\, dy$$

$$= (v_1 \int_0^\infty e^{-((v_1^\alpha - a)x)}\, dx)(v_2 \int_0^\infty e^{-(v_2^\alpha - b)y}\, dy)$$

By KUSHATR 1st integral formula

$=(\frac{v_1}{v_1^\alpha - a})(\frac{v_2}{v_2^\alpha - b})$ $=\frac{v_1 v_2}{(v_1^\alpha - a)(v_2^\alpha - b)}$

$$\therefore K_2(\exp(ax+by)) = \frac{v_1 v_2}{(v_1^\alpha - a)(v_2^\alpha - b)}$$

Formula 3)

If $f(x,y)$=*exp*$\big(i(ax+by)\big)$

$= K_2$(exp(i$(ax +$
$by)$)=$v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)}\, e^{i(ax+by)}\, dx\, dy$

$$= v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha - iax - iby)}\, dx\, dy$$

$$= v_1 v_2 \int_0^\infty \int_0^\infty e^{-[(v_1^\alpha - ia)x + (v_2^\alpha y - ib)y]}\, e^{(ax+by)}\, dx\, dy$$

$$= v_1 v_2 \int_0^\infty e^{-((v_1^\alpha - ia)x)} \int_0^\infty e^{-(v_2^\alpha - ib)y}\, dx\, dy$$

$$= (v_1 \int_0^\infty e^{-((v_1^\alpha - ia)x)}\, dx)(v_2 \int_0^\infty e^{-(v_2^\alpha - ib)y}\, dy)$$

By KUSHATR 1st integral formula

$=(\frac{v_1}{v_1^\alpha - ia})(\frac{v_2}{v_2^\alpha - ib})$ $=\frac{v_1 v_2}{(v_1^\alpha - ia)(v_2^\alpha - ib)}$

$$\therefore\ K_2(\exp(i(ax+by)) = \frac{v_1 v_2}{(v_1^\alpha - ia)(v_2^\alpha - ib)}$$

Formula 4)

If $f(x,y) = \cosh(ax+by)$

K_2\{cosh$(ax +$
$by)$\}=$v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)} \cosh(ax+by)\, dx\, dy$

$$=v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)} \left(\frac{e^{(ax+by)} + e^{-(ax+by)}}{2}\right) dx\, dy$$

$=\frac{1}{2} v_1 v_2 \int_0^\infty \int_0^\infty (e^{-(v_1^\alpha x + v_2^\alpha y)}\, e^{(ax+by)} +$
$e^{-(v_1^\alpha x + v_2^\alpha y)} e^{-(ax+by)})\, dx\, dy$

$=\frac{1}{2}(v_1 v_2 \int_0^\infty \int_0^\infty (e^{-(v_1^\alpha x + v_2^\alpha y)}\, e^{(ax+by)}\, dx\, dy +$
$v_1 v_2 \int_0^\infty \int_0^\infty (e^{-(v_1^\alpha x + v_2^\alpha y)}\, e^{-(ax+by)}\, dx\, dy)$

$=\frac{1}{2}(K_2\{\exp(ax+by)\}+K_2\{\exp(-(ax+by))\})$

By above 2nd formula

$=\frac{1}{2}\left(\frac{v_1v_2}{(v_1^\alpha-a)(v_2^\alpha-b)}+\frac{v_1v_2}{(v_1^\alpha+a)(v_2^\alpha+b)}\right)$

$$\therefore\ K_2\{\cosh(ax+by)\} = \frac{1}{2}\left(\frac{v_1v_2}{(v_1^\alpha-a)(v_2^\alpha-b)}+\frac{v_1v_2}{(v_1^\alpha+a)(v_2^\alpha+b)}\right)$$

Formula 5)

If f(x,y)=sinh$(ax+by)$

$K_2\{\sinh(ax+by)\}= v_1v_2\int_0^\infty\int_0^\infty e^{-(v_1^\alpha x+v_2^\alpha y)}\sinh(ax+by)\,dx\,dy$

$=v_1v_2\int_0^\infty\int_0^\infty e^{-(v_1^\alpha x+v_2^\alpha y)}\left(\frac{e^{(ax+by)}-e^{-(ax+by)}}{2}\right)dx\,dy$

$=\frac{1}{2}v_1v_2\int_0^\infty\int_0^\infty(e^{-(v_1^\alpha x+v_2^\alpha y)}e^{(ax+by)}-e^{-(v_1^\alpha x+v_2^\alpha y)}e^{-(ax+by)})\,dx\,dy$

$=\frac{1}{2}(v_1v_2\int_0^\infty\int_0^\infty(e^{-(v_1^\alpha x+v_2^\alpha y)}e^{(ax+by)}\,dx\,dy-v_1v_2\int_0^\infty\int_0^\infty(e^{-(v_1^\alpha x+v_2^\alpha y)}e^{-(ax+by)}\,dx\,dy)$

$=\frac{1}{2}(K_2\{\exp(ax+by)\}-K_2\{\exp(-(ax+by))\})$

By above 2nd formula

$=\frac{1}{2}\left(\frac{v_1v_2}{(v_1^\alpha-a)(v_2^\alpha-b)}-\frac{v_1v_2}{(v_1^\alpha+a)(v_2^\alpha+b)}\right)$

$$\therefore\ K_2\{\sinh(ax+by)\} = \frac{1}{2}\left(\frac{v_1v_2}{(v_1^\alpha-a)(v_2^\alpha-b)}-\frac{v_1v_2}{(v_1^\alpha+a)(v_2^\alpha+b)}\right)$$

Formula 6)

If $f(x,y)=\cos(ax+by)$

$K_2\{\cos(ax+by)\}=v_1v_2\int_0^\infty\int_0^\infty e^{-(v_1^\alpha x+v_2^\alpha y)}\cos(ax+by)\,dx\,dy$

$=v_1v_2\int_0^\infty\int_0^\infty e^{-(v_1^\alpha x+v_2^\alpha y)}\left(\frac{e^{i(ax+by)}+e^{-i(ax+by)}}{2}\right)dx\,dy$

$=\frac{1}{2}v_1v_2\int_0^\infty\int_0^\infty(e^{-(v_1^\alpha x+v_2^\alpha y)}e^{i(ax+by)}+e^{-(v_1^\alpha x+v_2^\alpha y)}e^{-i(ax+by)})\,dx\,dy$

$=\frac{1}{2}(v_1v_2\int_0^\infty\int_0^\infty(e^{-(v_1^\alpha x+v_2^\alpha y)}e^{i(ax+by)}\,dx\,dy+v_1v_2\int_0^\infty\int_0^\infty(e^{-(v_1^\alpha x+v_2^\alpha y)}e^{-i(ax+by)}\,dx\,dy)$

$=\frac{1}{2}(K_2\{\exp(i(ax+by))\}+K_2\{\exp(-\mathrm{i}(ax+by))\})$

By above 2nd formula

$=\frac{1}{2}\left(\frac{v_1v_2}{(v_1^\alpha-ia)(v_2^\alpha-ib)}+\frac{v_1v_2}{(v_1^\alpha+ia)(v_2^\alpha+ib)}\right)$

$$\therefore\ K_2\{\cos(ax+by)\} = \frac{1}{2}\left(\frac{v_1v_2}{(v_1^\alpha-ia)(v_2^\alpha-ib)}+\frac{v_1v_2}{(v_1^\alpha+ia)(v_2^\alpha+ib)}\right)$$

Formula 7)

If $f(x,y)=\sin(ax+by)$

$K_2\{\sin(ax+by)\}=v_1v_2\int_0^\infty\int_0^\infty e^{-(v_1^\alpha x+v_2^\alpha y)}\sin(ax+by)\,dx\,dy$

$=v_1v_2\int_0^\infty\int_0^\infty e^{-(v_1^\alpha x+v_2^\alpha y)}\left(\frac{e^{i(ax+by)}-e^{-i(ax+by)}}{2i}\right)dx\,dy$

$=\frac{1}{2i}v_1v_2\int_0^\infty\int_0^\infty(e^{-(v_1^\alpha x+v_2^\alpha y)}e^{i(ax+by)}-e^{-(v_1^\alpha x+v_2^\alpha y)}e^{-i(ax+by)})\,dx\,dy$

$=\frac{1}{2i}(v_1v_2\int_0^\infty\int_0^\infty(e^{-(v_1^\alpha x+v_2^\alpha y)}e^{i(ax+by)}\,dx\,dy-v_1v_2\int_0^\infty\int_0^\infty(e^{-(v_1^\alpha x+v_2^\alpha y)}e^{-i(ax+by)}\,dx\,dy)$

$=\frac{1}{2i}(K_2\{\exp(i(ax+by))\}-K_2\{\exp(-\mathrm{i}(ax+by))\})$

By above 2nd formula

$=\frac{1}{2i}\left(\frac{v_1v_2}{(v_1^\alpha-ia)(v_2^\alpha-ib)}-\frac{v_1v_2}{(v_1^\alpha+ia)(v_2^\alpha+ib)}\right)$

Formula 8)

If $f(x,y)$=$(xy)^n$,n>0

$K_2\{(xy)^n\}=v_1v_2\int_0^\infty\int_0^\infty e^{-(v_1^\alpha x+v_2^\alpha y)}(xy)^n\,dx\,dy$

$=v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x)}\, e^{-(v_2^\alpha y)}\, x^n y^n\, dx\, dy$

$=(v_1 \int_0^\infty e^{-((v_1^\alpha x)} x^n\, dx)(v_2 \int_0^\infty e^{-(v_2^\alpha y)} y^n\, dy)$

By definition ofKUSHARE transform

$= K\{x^n\} K\{y^n\}$

$=\left(\frac{\Gamma(n+1)}{v_1^{\alpha((n+1)-1)}}\right)\left(\frac{\Gamma(n+1)}{v_2^{\alpha((n+1)-1)}}\right)$

$=\frac{(\Gamma(n+1))^2}{(v_1 v_2)^{\alpha((n+1)-1)}}$

$$\therefore K_2\{(xy)^n\} = \frac{(\Gamma(n+1))^2}{(v_1 v_2)^{\alpha((n+1)-1)}}$$

Formula 9) If $f(x,y)$=$x^m y^n$, m>0, n>0

$$K_2\{x^m y^n\} = v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)}\, x^m y^n\, dx\, dy$$

$=v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x)}\, e^{-(v_2^\alpha y)}\, x^m y^n\, dx\, dy$

$=(v_1 \int_0^\infty e^{-((v_1^\alpha x)} x^m\, dx)(v_2 \int_0^\infty e^{-(v_2^\alpha y)} y^n\, dy)$

By definition ofKUSHARE transform

$= K\{x^m\} K\{y^n\}$

$=\left(\frac{\Gamma(m+1)}{v_1^{\alpha((m+1)-1)}}\right)\left(\frac{\Gamma(n+1)}{v_2^{\alpha((n+1)-1)}}\right)$

$=\frac{\Gamma(m+1)\Gamma(n+1)}{v_1^{\alpha((m+1)-1)} v_2^{\alpha((n+1)-1)}}$

$$\therefore K_2\{x^m y^n\} = \frac{\Gamma(m+1)\Gamma(n+1)}{v_1^{\alpha((m+1)-1)} v_2^{\alpha((n+1)-1)}}$$

Double Kushare Integral transform of some functions

Function $f(x,y)$	Double New Integral Transform$K_2[f(x,y)]$
1	$\frac{v_1 v_2}{v_1^\alpha v_2^\alpha}$
$\exp(ax + by)$	$\frac{v_1 v_2}{(v_1^\alpha - a)(v_2^\alpha - b)}$
$\exp(i(ax + by))$	$\frac{v_1 v_2}{(v_1^\alpha - ia)(v_2^\alpha - ib)}$
$\cosh(ax + by)$	$\frac{1}{2}\left(\frac{v_1 v_2}{(v_1^\alpha - a)(v_2^\alpha - b)} + \frac{v_1 v_2}{(v_1^\alpha + a)(v_2^\alpha + b)}\right)$
$\sinh(ax + by)$	$\frac{1}{2}\left(\frac{v_1 v_2}{(v_1^\alpha - a)(v_2^\alpha - b)} - \frac{v_1 v_2}{(v_1^\alpha + a)(v_2^\alpha + b)}\right)$
$\cos(ax + by)$	$\frac{1}{2}\left(\frac{v_1 v_2}{(v_1^\alpha - ia)(v_2^\alpha - ib)} + \frac{v_1 v_2}{(v_1^\alpha + ia)(v_2^\alpha + ib)}\right)$
$\sin(ax + by)$	$\frac{1}{2i}\left(\frac{v_1 v_2}{(v_1^\alpha - ia)(v_2^\alpha - ib)} - \frac{v_1 v_2}{(v_1^\alpha + ia)(v_2^\alpha + ib)}\right)$
$(xy)^n$, n>0	$\frac{(\Gamma(n+1))^2}{(v_1^\alpha v_2^\alpha)^{\alpha((n+1)-1)}}$
$x^m y^n$, m>0, n>0	$\frac{\Gamma(m+1)\Gamma(n+1)}{v_1^{\alpha((m+1)-1)} v_2^{\alpha((n+1)-1)}}$

Theorem 1: Let $f(x,y)$ be a function of two variables. If the first ordered partial derivative $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exists and $f(0,y)$ be given.$v_{1,} v_2, v_1^\alpha, v_2^\alpha$ are positive real functions then

$K_2[\frac{\partial f}{\partial x}$ (x, y)] = -v_1K{$f(0,y)$}+$v_1^\alpha K_2\{f(x,y)\}$

Where K{$f(0,y)$} is the new KUSHARE integral transform of the $f(0,y)$

Proof

$K_2[\frac{\partial f}{\partial x}$ (x, y)] = $v_1 v_2 \int_0^\infty \int_0^\infty \frac{\partial f}{\partial x} e^{-(v_1^\alpha x + v_2^\alpha y)}\, dx\, dy$

$=v_1 v_2 \int_0^\infty (\int_0^\infty \frac{\partial f}{\partial x} e^{-(v_1^\alpha x)} dx)\, e^{-(v_2^\alpha y)}\, dy$

$=v_1 v_2 \int_0^\infty (e^{-(v_1^\alpha x)} \int_0^\infty \frac{\partial f}{\partial x} dx -$
$\int_0^\infty (-v_1^\alpha e^{-(v_2^\alpha y)} \int_0^\infty \frac{\partial f}{\partial x} dx) dx) e^{-(v_2^\alpha y)} dy$

$$= v_1 v_2 \int_0^\infty (-f(0,y) + v_1^\alpha \int_0^\infty e^{-(v_1^\alpha x)} f(x,y) dx)\, e^{-(v_2^\alpha y)} dy$$

$=-v_1 v_2 \int_0^\infty f(0,y) e^{-(v_2^\alpha y)} dy +$
$v_1 v_2 v_1^\alpha \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)} f(x,y) dx\, dy$

$=-v_1 (v_2 \int_0^\infty f(0,y) e^{-(v_2^\alpha y)} dy) +$
$v_1^\alpha (v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)} f(x,y) dx\, dy)$

By definition of second KUSHARE transform

= $-v_1$ K$\{f(0,y)\}$+$v_1^\alpha K_2\{f(x,y)\}$

Theorem 2: Let $f(x,y)$ be a function of two variables. If the first ordered partial derivative $\frac{\partial f}{\partial x}$ *and* $\frac{\partial f}{\partial y}$ exists and $f(x,0)$ be the given in condition.$v_{1,} v_2, v_1^\alpha, v_2^\alpha$ are positive real functions then

$K_2[\frac{\partial f}{\partial y}$ (x, y)] = $-v_2$ K$\{f(x,0)\}$+$v_2^\alpha K_2\{f(x,y)\}$

where K$\{f(x,0)\}$ is the new KUSHARE integral transform of the $f(x,0)$

Proof.

$K_2[\frac{\partial f}{\partial y}$ (x, y)] = $v_1 v_2 \int_0^\infty \int_0^\infty \frac{\partial f}{\partial y} e^{-(v_1^\alpha x + v_2^\alpha y)} dx\, dy$

$=v_1 v_2 \int_0^\infty (\int_0^\infty \frac{\partial f}{\partial y} e^{-(v_2^\alpha y)} dy)\, e^{-(v_1^\alpha x)} dx$

$= v_1 v_2 \int_0^\infty (e^{-(v_2^\alpha y)} \int_0^\infty \frac{\partial f}{\partial y} dy -$
$\int_0^\infty (-v_2^\alpha e^{-(v_2^\alpha y)} \int_0^\infty \frac{\partial f}{\partial y} dy) dy) e^{-(v_1^\alpha x)} dx$

$$= v_1 v_2 \int_0^\infty (-f(x,0) + v_2^\alpha \int_0^\infty e^{-(v_2^\alpha y)} f(x,y) dy)\, e^{-(v_1^\alpha x)} dx$$

$=-v_1 v_2 \int_0^\infty f(x,0) e^{-(v_1^\alpha x)} dx +$
$v_1 v_2 v_2^\alpha \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)} f(x,y) dx\, dy$

$=-v_2 (v_1 \int_0^\infty f(x,0) e^{-(v_1^\alpha x)} dx) +$
$v_2^\alpha (v_1 v_2 \int_0^\infty \int_0^\infty e^{-(v_1^\alpha x + v_2^\alpha y)} f(x,y) dx\, dy)$

By definition of second KUSHARE transform ∴ $K_2[\frac{\partial f}{\partial y}$ (x, y)] $= -v_2\, K\{f(x,0)\} + v_2^\alpha K_2\{f(x,y)\}$

IV. APPLICATIONS

in this section we solve initial boundary value problems.

Example: Solve the partial differential equation $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$ with the initial conditions

$$f(0,y) = y, f(x,0) = x$$

Solution: Let $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$ Applying double new general integral transform we get

$K_2[\frac{\partial f}{\partial x}$ (x, y)] = $K_2[\frac{\partial f}{\partial y}$ (x,y)]

$\Rightarrow$ $-v_1$ K$\{f(0,y)\}$+$v_1^\alpha K_2\{f(x,y)\}$ = $-v_2$ K$\{f(x,0)\}$+$v_2^\alpha K_2\{f(x,y)\}$

$$\Rightarrow -v_1 \text{K}\{y\} + v_1^\alpha K_2\{f(x,y)\} = -v_2 \text{K}\{x\} + v_2^\alpha K_2\{f(x,y)\}$$

$$\Rightarrow -v_1 \frac{v_2}{v_2^{2\alpha}} + v_1^\alpha K_2\{f(x,y)\} = -v_2 \frac{v_1}{v_1^{2\alpha}} + v_2^\alpha K_2\{f(x,y)\}$$

$$\Rightarrow K_2\{f(x,y)\}(v_1^\alpha - v_2^\alpha) = \frac{v_1 v_2}{v_2^{2\alpha}} - \frac{v_1 v_2}{v_1^{2\alpha}}$$

$$\Rightarrow K_2\{f(x,y)\}(v_1^\alpha - v_2^\alpha) = v_1 v_2 \left(\frac{1}{v_2^{2\alpha}} - \frac{1}{v_1^{2\alpha}}\right)$$

$$\Rightarrow K_2\{f(x,y)\}(v_1^\alpha - v_2^\alpha) = v_1 v_2 \left(\frac{v_1^{2\alpha} - v_2^{2\alpha}}{v_2^{2\alpha} v_1^{2\alpha}}\right)$$

$$\Rightarrow K_2\{f(x,y)\} = \left(\frac{v_1 v_2}{v_1^{2\alpha} v_2^{2\alpha}}\right)(v_1^\alpha + v_2^\alpha)$$

$$\Rightarrow K_2\{f(x,y)\} = \left(\frac{v_1 v_2}{v_1^{\alpha} v_2^{2\alpha}}\right) + \left(\frac{v_1 v_2}{v_1^{2\alpha} v_2^{\alpha}}\right)$$

$$\Rightarrow f(x,y) = \text{x} + y$$

CONCLUSION

In this paper we have successfully developed double Kushare intrgral transform.

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