

Modeling Gases Transfer Between Placenta and Fetus Across the Umbilical Cord

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Abstract- This study presents a proposed mathematical model gases exchange across umbilical cord to provide a system of equations whose solution can be analyzed for equilibrium and stability. In this paper we use qualitative theory of differential equations to develop a four compartmental Mathematical model that takes into Umbilical cord by consideration exchange of gases, nutrients and wastes through the umbilical cord.

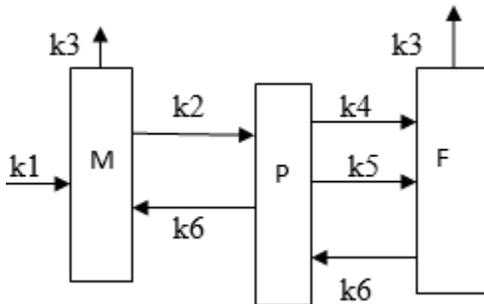
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Indexed Terms- placenta, umbilical cord

I. INTRODUCTION

The model so far developed in nutrient and waste exchange in the placenta was a system of linear differential equations as shown;

$$\begin{aligned} \frac{dM}{dt} &= k_1 + k_6 P - (k_3 + k_2) M \\ \frac{dP}{dt} &= k_2 M + k_6 F - (k_4 + k_5 + k_6) P \\ \frac{dF}{dt} &= (k_4 + k_5) P - (k_3 + k_6) F \end{aligned}$$



Where:

M = the total calories the mother has (calories)

F = the total calories the fetus has (calories)

P = the placenta (the interface)

k1 = the number of calories the pregnant woman will intake in a given day (calories/day)

k2 = the rate of calorie transfer between mother and the fetus (1/day)

k3 = the rate of calorie burn-off and waste (1/day)

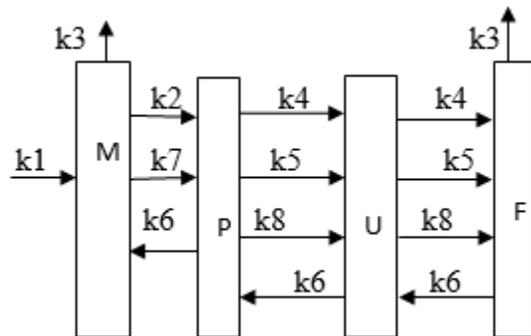
k4 = the rate of calorie transfer between mother and fetus partially consumed (1/day)

k5 = the rate of calorie transfer between mother and fetus metabolized (1/day)

k6 = the amount of waste product transferred from fetus to the mother through the placenta

II. PRELIMINARY

There is need to develop a new model which takes into account the Umbilical cord. The importance of Umbilical is to carry the fetus blood back and forth, between the fetus and the placenta as it delivers nutrients and oxygen to the fetus and removes the fetus waste products.



Where:

M = the total calories the mother has (calories)

F = the total calories the fetus has (calories)

P = the placenta (the interface)

k₁ = the number of calories the pregnant woman will intake in a given day (calories/day)

k₂ = the rate of calorie transfer between mother and the fetus (1/day)

k₃ = the rate of calorie burn-off and waste (1/day)

k₄ = the rate of calorie transfer between mother and fetus partially consumed (1/day)

k₅ = the rate of calorie transfer between mother and fetus metabolized (1/day)

k₆ = the amount of waste product transferred from fetus to the mother through the placenta and umbilical cord (gases included)

k₇ = the amount of gases transferred from mother to the fetus through the placenta and umbilical cord

k₈ = the rate of gases transfer between placenta and fetus across the umbilical cord (filtered)

CONCLUSION

The model includes the interface between the mother, foetus and Umbilical cord in which nutrients, gases and waste Products are also trans- ported from foetus to mother.

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