Inductance Matrix of a Transfer Field Machine (TFM) And an Induction Machine (IM): A Comparison

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Abstract- This paper presents a comparative analysis of a transfer field machine (TFM) and a poly-phase induction machine (IM) with central focus on the inductance matrix of both machines. The two machines belong to two different classes of machine and quite different in physical configuration. In this analysis, the self-inductance matrix of the two machines is derived and both shown to be independent of the rotor angular position. However, the mutual coupling inductance in both cases are dependent on rotor angular position which varies with time. For the transfer field machine, in addition to rotor angle dependence, it also depends on the difference between the direct- and quadrature-axes reactances. In both machines, when state variables namely voltage and flux linkage equations, are transformed to arbitrary qdo reference frame, a new set of voltage and flux linkage equations are obtained with inductance values, no longer dependent on the rotor angular position that varies with time; and this of course is of great advantage in the analysis of both machines.

Indexed Terms- Self-inductance, Mutual-inductance, Inductance Matrix, Coupling, Angular position, Comparative Analysis.

I. INTRODUCTION

The theory of induction machine is old and well known. An induction machine consists essentially of two major parts, the stator and the rotor. When an a.c voltage is impressed on the terminals of the stator windings, a rotating magnetic field is set up. This rotating magnetic field produces an electromotive force (e.m.f) in the rotor by electromagnetic induction (transformer action) which in turn, circulate current in the rotor usually short-circuited. This current circulating in the short-circuited rotor, produces a rotating magnetic field which now interact with the rotating magnetic field already established in the stator. This interaction produces a torque which is responsible for the rotation of the machine.

Induction machine is also known as the asynchronous machine which derives from the fact that the rotor magnetic field is always lagging the stator magnetic field. The difference is called the slip, and it is a fundamental characteristic in the operation of an induction machine. An induction machine when it operates below synchronous speed, is a motor while it is a generator when it operates above the synchronous speed. In fact, induction machines are mostly used as motors.

The induction motor is used in a wide variety of applications as a means of converting electric power to mechanical work. It is without doubt, the workhorse of the electric power industry. Pump, steel mill and hoist drives are but few applications of large multiphase induction motors. On a smaller scale, the single-phase servo motor is used extensively in position-follow-up control systems and single – phase induction motors are widely used in household appliances as well as hand and bench tools [1].

The transfer-field machine (TFM) is structurally basically a reluctance machine. It differs however from the simple reluctance machine in two important respects namely: -

(a) it has two sets of windings instead of one
(b) each winding has a synchronous reactance which is independent of rotor position whereas the winding reactance of a single reluctance machine varies cyclically [2].

The TF machine configuration has two stator windings in each machine element known as main and auxiliary windings. The main windings are connected in series while the auxiliary windings are connected in series but transposed between the two machine sections. There are no windings on the rotors of either of the composite machines. This machine induces negative sequence emfs of frequency $(2S – 1)\omega_o$ in the auxiliary windings which will in turn circulate a current excluded from the supply.

The interaction of the main and auxiliary winding magneto motive forces (mmfs), will produce an interference wave with beat frequency, ω, which is equal to the rotor frequency. Hence a reluctance torque is developed in the rotor as a result of its interaction with the interference wave and this causes the rotor and hence the machine to rotate (turn).

And so, a transfer-field machine is an energy converter and like the induction machine, is asynchronous and self-starting. The transfer – field machine is very useful in control systems, electrical gear, low speed drives etc. Again, its auxiliary winding terminals which will act as the rotor conductors in normal induction machine is available without requiring slip rings or current collection gears. It can also be used to supply a d.c load through rectifiers, a function which has not been performed satisfactorily by induction motors because the output waveforms of induction motors tend to be increasingly distorted as the load current increases. Also, it is capable of survival in a harsh environment [3].

II. PHYSICAL CONFIGURATION OF A TRANSFER FIELD MACHINE (TFM)

The transfer field machine (TFM) comprises a two-stack machine in which the rotor is made up of two identical equal halves whose pole axes are $\pi/2$ radians out of phase in space. They are housed in their respective induction motor type stators. There are no windings in the rotor. The stator has two physically isolated but magnetically coupled identical windings known as the main and auxiliary windings. The axes of the main windings are the same in both halves of the machine whereas the axes of the auxiliary windings are transposed in passing from one half of the machine to the other. Both sets of winding are distributed in the stator slots and occupy the same slots for perfect coupling and have the same number of poles. The two sets of winding of the transfer field machine are essentially similar and may be connected in parallel which will of course double its output. The schematic diagram of a transfer field machine (TFM) is as illustrated in figure 1.0

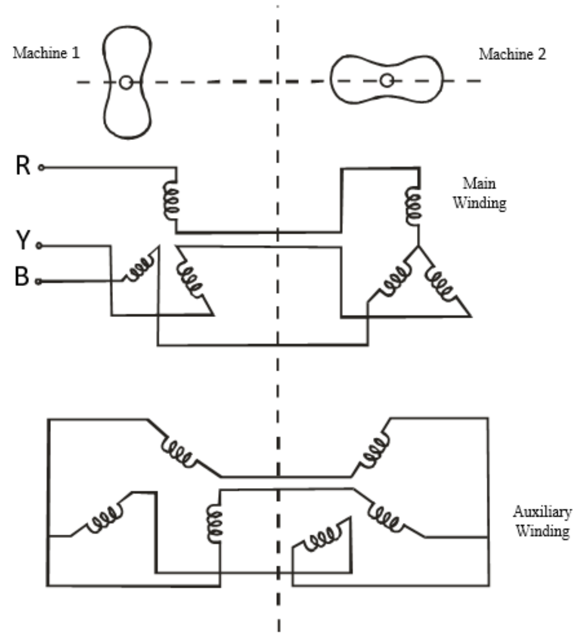


Fig1.0: Connection diagram for a transfer field machine (TFM)

III. INDUCTANCE MATRIX OF A TRANSFER FIELD MACHINE (TFM)

3.1 TFM Self - and Mutual – inductances

The rotor of a TFM is of salient poles without conductors and as a result, its mmfs are always directed along the d- and q- axes. Also, the direction of the resultant mmf of the stator windings relative to d- and q- axes will vary with the power factor. A common approach to handling the magnetic effect of the stators resultant mmf is to resolve it along the d - and q - axes.

Let us consider the magnetic effect of a current flowing in one phase and let this phase be denoted by “a”. Hence the resolved components of the a-phase mmf, Fa, will produce the flux components;

$$\left.\begin{aligned} \phi_d &= p_{ed}\, F_a \sin\theta_r \\ \phi_q &= p_{eq}\, F_a \cos\theta_r \end{aligned}\right\} \quad (1)$$

along the d- and q-axes respectively.

Where; Pe = permeance.

The flux linkage of these resolved components with the a-phase winding is;

$\lambda_{aa}=N_s(\phi_d sin\theta_r + \phi_q cos\theta_r)$ (2)

Substituting equation (1) into equation (2), will yield;

$\lambda_{aa} = N_s F_a(p_e d\theta_r + p_e q cos^2\theta_r)$

$\lambda_{aa} = N_s F_a\{ \frac{p_e d+p_e q}{2} - \frac{p_e d-p_e q}{2} cos2\theta_r\}$ (3)

In a similar manner, the linkage of the flux components, ϕ_d and ϕ_q by the b-phase winding that is $\frac{2\pi}{3}$ ahead may be written as;

$\lambda_{ba} = N_s F_a\{p_{e^d} sin\theta_r \sin(\theta_r - 2\pi/3) +$

$p_{e^q} cos\theta_r cos(2\theta_r - 2\pi/3)\}$

$\therefore \lambda_{ba} =$ Ns $Fa\left\{\frac{p_e d + p_e q}{4} - \frac{p_e d - p_e q}{4} cos2(\theta_r - \frac{\pi}{3})\right\}$
(4)

(

We can deduce based on the functional relationship of λ_{aa} with the rotor angle, θ_r, that the self-inductance of the stator a-phase winding, excluding the leakage inductance, has the form;

$L_{aa}=L_o-L_{ms}cos2\theta_r$ (5)

Where;

$L_o= \frac{L_{md}+L_{mq}}{2} and L_{ms}=\frac{L_{md}-L_{mq}}{2}$

The self-inductances of the b- and c- phases, L_{bb} and L_{cc}, are similar to that of L_{aa} but with θ_r replaced by $(\theta_r - \frac{\pi}{3})$ and $(\theta_r + \frac{\pi}{3})$ respectively.

Similarly, it can be deduced from equation (4) that the mutual inductance between the a-and b-phases of the stator is of the form;

$L_{ab}=L_{ba}=\frac{-L_o}{2} - L_{ms}cos2(\theta_r - \frac{\pi}{3})$ (6)

Again the mutual inductances L_{bc} and L_{ac} can be obtained by replacing θ_r with $(\theta_r + \frac{\pi}{3})$ and $(\theta_r + \frac{2\pi}{3})$ respectively.

For this analysis, the upper case subscripts – A, B, C will be associated with the parameters of the main windings while the lower case subscripts – a,b,c will be associated with the auxiliary windings.

For a three-phase machine like the TFM, the voltage equation for the main (stator) winding is;

$$\left.\begin{aligned} V_{ABC} &= r_{ABC}i_{ABC} + P\lambda_{ABC} \\ V_{abc} &= r_{abc}i_{abc} + P\lambda_{abc} \end{aligned}\right\} \quad (7)$$

where;

$P = \frac{d}{dt}$

λ = flux linkage

r_{ABC} = diag $([r_A \; r_B \; r_C])$

r_{abc} = diag $([r_a \; r_b \; r_c])$

The flux linkages in stator reference frame are expressed as;

$$\begin{bmatrix} \lambda_{ABC} \\ \lambda_{abc} \end{bmatrix} = \begin{bmatrix} L_{xx} & L_{xy} \\ L_{yx} & L_{yy} \end{bmatrix} \begin{bmatrix} i_{ABC} \\ i_{abc} \end{bmatrix} \quad (8)$$

where;

L_{xx} = self-inductance for main windings of TFM

L_{yy} = self-inductance for auxiliary windings of TFM

Lxy = Lyx = mutual inductance between main and auxiliary windings

The inductance matrices terms L_{xx}, L_{xy}, L_{yx} and L_{yy} are obtained from inductance submatrices $L_{11}, L_{12}, L_{21} and L_{22}$ for machine 1 and machine 2.

Now for machine 1, the self-inductance submatrix for the main winding is;

$$L_{11}^{m/c1} = \begin{bmatrix} L_{1s} + L_o - L_{ms}\cos 2\theta_r & -\tfrac{1}{2}L_o - L_{ms}\cos 2\left(\theta_r - \tfrac{\pi}{3}\right) & -\tfrac{1}{2}L_o - L_{ms}\cos 2\left(\theta_r + \tfrac{\pi}{3}\right) \\ -\tfrac{1}{2}L_0 - L_{ms}\cos 2\left(\theta_r - \tfrac{\pi}{3}\right) & L_{ls} + L_o - L_{ms}\cos 2\left(\left(\theta_r - \tfrac{\pi}{3}\right)\right) & -\tfrac{1}{2}L_o - L_{ms}\cos 2\theta_r \\ -\tfrac{1}{2}L_o - L_{ms}\cos 2\left(\theta_r + \tfrac{\pi}{3}\right) & -\tfrac{1}{2}L_o - L_{ms}\cos 2\theta_r & L_{ls} + L_o - L_{ms}\cos 2\left(\theta_r + \tfrac{\pi}{3}\right) \end{bmatrix} \quad (9)$$

For machine 2, the Self-inductance submatrix for the main winding is;

$$L_{11}^{m/c2} = \begin{bmatrix} L_{1s} + L_o + L_{ms}\cos 2\theta_r & -\tfrac{1}{2}L_o + L_{ms}\cos 2\left(\theta_r - \tfrac{\pi}{3}\right) & -\tfrac{1}{2}L_o + L_{ms}\cos 2\left(\theta_r + \tfrac{\pi}{3}\right) \\ -\tfrac{1}{2}L_0 + L_{ms}\cos 2\left(\theta_r - \tfrac{\pi}{3}\right) & L_{ls} + L_o + L_{ms}\cos 2\left(\theta_r - \tfrac{\pi}{3}\right) & -\tfrac{1}{2}L_o + L_{ms}\cos 2\theta_r \\ -\tfrac{1}{2}L_o + L_{ms}\cos 2\left(\theta_r + \tfrac{\pi}{3}\right) & -\tfrac{1}{2}L_o + L_{ms}\cos 2\theta_r & L_{ls} + L_o + L_{ms}\cos 2\left(\theta_r + \tfrac{\tau}{3}\right) \end{bmatrix} \quad (10)$$

The self-inductance matrix, L_{xx}, for the main winding of TF machine is obtained by adding the self-inductance submatrix, L_{11} of machine 1 and the self-inductance submatrix, L_{11} of machine 2. And this addition will yield;

$$L_{xx} = \begin{bmatrix} 2L_{ls} + 2L_o & -L_o & -L_o \\ -L_o & 2L_{ls} + 2L_o & -L_o \\ -L_o & -L_o & 2L_{ls} + 2L_o \end{bmatrix} \quad (11)$$

Where:

$L_{ls} =$ Leakage inductance of main (stator) winding

$$L_o = \frac{L_{md} + L_{mq}}{2} \quad (12)$$

Substituting equation (12) into equation (11), gives;

$$L_{xx} = \begin{bmatrix} 2L_{1s} + L_{md} + L_{mq} & -\tfrac{1}{2}\left(L_{md} + L_{mq}\right) & -\tfrac{1}{2}\left(L_{md} + L_{mq}\right) \\ -\tfrac{1}{2}\left(L_{md} + L_{mq}\right) & 2L_{ls} + L_{md} + L_{mq} & -\tfrac{1}{2}\left(L_{md} + L_{mq}\right) \\ -\tfrac{1}{2}\left(L_{md} + L_{mq}\right) & -\tfrac{1}{2}\left(L_{md} + L_{mq}\right) & 2L_{ls} + L_{md} + L_{mq} \end{bmatrix} \quad (13)$$

From equation (11), it is very evident that the self-inductance matrix, L_{xx}, of the main winding, is the sum of self and mutual inductances of machine 1 and machine 2 respectively. The mutual inductance between the main and auxiliary winding, L_{xy}, is obtained by adding the mutual inductance submatrix, L_{12} for machine 1 to the mutual inductance submatrix, L_{12} for machine 2. These mutual inductance submatrices are;

$$L_{12}^{M/C1} = \begin{bmatrix} L_{1s} + L_o - L_{ms}\cos 2\theta_r & -\frac{1}{2}L_o - L_{ms}(2\theta_r - \alpha) & -\frac{1}{2}L_o - L_{ms}\cos(2\theta_r + \alpha) \\ -\frac{1}{2}L_o - L_{ms}\cos(2\theta_r - \alpha) & L_{ls} + L_o - L_{ms}\cos(2\theta_r + \alpha) & -\frac{1}{2}L_o - L_{ms}\cos 2\theta_r \\ -\frac{1}{2}L_o - L_{ms}\cos(2\theta_r + \alpha) & -\frac{1}{2}L_o - L_{ms}\cos 2\theta_r & L_{ls} + L_o - L_{ms}\cos(2\theta_r - \alpha) \end{bmatrix} \quad (14)$$

$$L_{12}^{M/C2} = \begin{bmatrix} L_{1s} + L_o + L_{ms}\cos 2\theta_r & -\frac{1}{2}L_o + L_{ms}(2\theta_r - \alpha) & -\frac{1}{2}L_o + L_{ms}\cos(2\theta_r + \alpha) \\ -\frac{1}{2}L_o + L_{ms}\cos(2\theta_r - \alpha) & L_{ls} + L_o + L_{ms}\cos(2\theta_r + \alpha) & -\frac{1}{2}L_o + L_{ms}\cos 2\theta_r \\ -\frac{1}{2}L_o + L_{ms}\cos(2\theta_r + \alpha) & -\frac{1}{2}L_o + L_{ms}\cos 2\theta_r & L_{ls} + L_o + L_{ms}\cos(2\theta_r - \alpha) \end{bmatrix} \quad (15)$$

Therefore, the mutual inductance between the main and auxiliary winding, L_{xy}, taken into account the transposition of the auxiliary winding, is;

$$L_{xy} = L_{12}^{M/C1} + xL_{12}^{M/C2} = \begin{bmatrix} -2L_{ms}\cos 2\theta_r & -2L_{ms}\cos(2\theta_r - \alpha) & -2L_{ms}\cos(2\theta_r + \alpha) \\ -2L_{ms}\cos(2\theta_r - \alpha) & -2L_{ms}\cos(2\theta_r + \alpha) & -2L_{ms}\cos 2\theta_r \\ -2L_{ms}\cos(2\theta_r + \alpha) & -2L_{ms}\cos 2\theta_r & -2L_{ms}\cos(2\theta_r - \alpha) \end{bmatrix}$$

Taking out a common factor $-2L_{ms}$, the mutual inductance, L_{xy} , becomes;

$$L_{xy} = -2L_{ms}\begin{bmatrix} \cos 2\theta_r & \cos(2\theta_r - \alpha) & \cos(2\theta_r + \alpha) \\ \cos(2\theta_r - \alpha) & \cos(2\theta_r + \alpha) & \cos 2\theta_r \\ \cos(2\theta_r + \alpha) & \cos 2\theta_r & \cos(2\theta_r - \alpha) \end{bmatrix}$$

But $L_{ms} = \dfrac{L_{md} - L_{mq}}{2}$

Hence $-2L_{ms} = -2\left\{\dfrac{L_{md} - L_{mq}}{2}\right\} = L_{mq} - L_{md}$ and if this is substituted into the expression for L_{xy}, we have that;

$$L_{xy} = L_{mq} - L_{md}\begin{bmatrix} \cos 2\theta_r & \cos(2\theta_r - \alpha) & \cos(2\theta_r + \alpha) \\ \cos(2\theta_r - \alpha) & \cos(2\theta_r + \alpha) & \cos 2\theta_r \\ \cos(2\theta_r + \alpha) & \cos 2\theta_r & \cos(2\theta_r - \alpha) \end{bmatrix} \quad (16)$$

Where; $\alpha = \frac{2\pi}{3}$ (120^0)

Now because the main and auxiliary windings in both machine 1 and machine 2 are identical in nature, it is clear to assume in this paper that $L_{xx} = L_{yy}$ and $L_{xy} = L_{yx}$. And for this reason, auxiliary winding parameters do not change values when they are referred to the main winding [4].

3.2 Transforming of TFM main winding (stator) quantities to arbitrary qdo reference frame.

The rotor of the TF machine is salient pole without winding conductors and as a result, its mmfs are always directed along the d- and q-axes. The consequence of this is that the qdo transformations can only be applied to the stator quantities. The main purpose of this transformation is to obtain constant inductances whose values will not depend on the rotor angular position that varies with time which is evident from equation (16).

(1) TFM Voltage equations in qdo reference frame:
For a three-phase machine like the TFM, the voltage equation for the main (stator) winding is;

$$\left.\begin{aligned} V_{ABC} &= r_{ABC}\, i_{ABC} + p\lambda_{ABC} \\ V_{abc} &= r_{abc}\, i_{abc} + p\lambda_{abc} \end{aligned}\right\} \qquad (17)$$

where;

$$P = {}^{d}\!/_{dt}$$

λ = Flux linkage

r_{ABC} = diag $\left(\left[r_A\ r_B\ r_C\right]\right)$

r_{abc} = diag $\left(\left[r_a\ r_b\ r_c\right]\right)$

applying the T_{qdo} (θ_r) to equation (17), gives;

$$V^{QDO} = T_{QDO}(\theta_r) r_{ABC}\, T_{QDO}^{-1}(\theta_r) I_{QDO} + T_{QDO}(\theta_r) \mathrm{P}\, T_{QDO}^{-1} \lambda_{QDO}\,. \qquad (18)$$

Where;

(i)

$$T_{QDO}(\theta_r) = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ Sin\theta_r & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

(ii)

$$T_{QDO}^{-1}(\theta_r) = \begin{bmatrix} \cos\theta_r & Sin\theta_r & 1 \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \qquad (19)$$

(iii) $P = \frac{d}{dt}$

Substituting the above expressions into equation (18) and solving, gives the voltage equations for the main winding as;

$$\left.\begin{aligned} V_Q &= ri_Q + \omega\lambda_D + p\lambda_Q \\ V_D &= ri_D - \omega\lambda_Q + p\lambda_D \\ V_O &= ri_O + p\lambda_O \end{aligned}\right\} \qquad (20)$$

While the voltage equations for the auxiliary windings become;

$$\left.\begin{aligned} V_q &= ri_q - (\omega_o - 2\omega)\lambda_d + p\lambda_q \\ V_d &= ri_d + (\omega_o - 2\omega)\lambda_q + p\lambda_d \\ V_o &= ri_o + p\lambda_o \end{aligned}\right\}. \qquad (21)$$

(2) TFM Flux linkage equation in qdo reference:
The flux linkage equation (equation 8) is rewritten in dqo frame as;

$$\begin{bmatrix} [\lambda_Q & \lambda_D & \lambda_O] \\ [\lambda_q & \lambda_d & \lambda_o] \end{bmatrix}^T = \begin{bmatrix} K_x L_{xx}(K_x)^{-1} & K_x L_{xy}(K_y)^{-1} \\ K_y L_{yx}(K_x)^{-1} & K_y L_{yy}(K_y)^{-1} \end{bmatrix} \begin{bmatrix} [I_Q\ I_D\ I_O] \\ [I_q\ I_d\ I_o] \end{bmatrix}$$

(22)

Where;

$$K_x = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta-\alpha) & \cos(\theta+\alpha) \\ \sin\theta & \sin(\theta-\alpha) & \sin(\theta+\alpha) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \qquad (23)$$

$$(K_x)^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos(\theta-\alpha) & \sin(\theta-\alpha) & 1 \\ \cos(\theta+\alpha) & \sin(\theta+\alpha) & 1 \end{bmatrix} \qquad (24)$$

$$K_y = \frac{2}{3}\begin{bmatrix} \cos\beta & \cos(\beta-\alpha) & \cos(\beta+\alpha) \\ \sin\beta & \sin(\beta-\alpha) & \sin(\beta+\alpha) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

(25)

$$(K_y)^{-1} = \begin{bmatrix} \cos\beta & \sin\beta & 1 \\ \cos(\beta-\alpha) & \sin(\beta-\alpha) & 1 \\ \cos(\beta+\alpha) & \sin(\beta+\alpha) & 1 \end{bmatrix} \qquad (26)$$

From equation (18) through equation (26);

$\alpha = \dfrac{2\pi}{3}$

β = Speed of rotation of arbitrary reference frame.

θ_r = Angular rotor position

T = Matrix transpose

Substituting equations (23 – 26) back into equation (22), the flux linkage equations can now be expressed as;

$$\left.\begin{aligned}
\lambda_Q &= (2L_l + L_{mq} + L_{md})I_Q - (l_{md} - L_{mq})I_q \\
\lambda_D &= (2L_l + L_{mq} + L_{md})I_D + (l_{md} - L_{mq})I_d \\
\lambda_O &= 2L_l I_O \\
\lambda_q &= (2L_l + L_{mq} + L_{md})I_q - (l_{md} - L_{mq})I_Q \\
\lambda_d &= (2L_l + L_{mq} + L_{md})I_d + (l_{md} - L_{mq})I_D \\
\lambda_o &= 2L_i I_o
\end{aligned}\right\} \quad (.27)$$

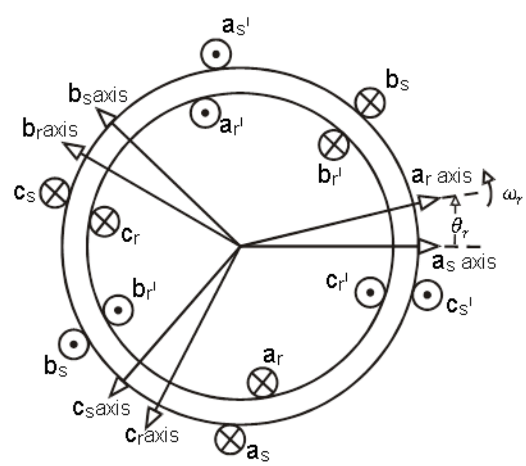
In its completeness, equation (27) can be rewritten as;

$$\begin{bmatrix} \lambda_Q \\ \lambda_D \\ \lambda_O \\ \lambda_q \\ \lambda_d \\ \lambda_o \end{bmatrix} = \begin{bmatrix} 2L_l + L_{mq} + L_{md} & 0 & 0 & -(L_{md} - L_{mq}) & 0 & 0 \\ 0 & .2L_L + L_{mq} + L_{md} & 0 & 0 & L_{md} - L_{mq} & 0 \\ 0 & 0 & 2L_L & 0 & 0 & 0 \\ -(L_{md} - L_{mq}) & 0 & 0 & 2L_L + L_{mq} + L_{md} & 0 & 0 \\ 0 & L_{md} - L_{mq} & 0 & 0 & 2L_L + L_{mq} + L_{md} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2L_L \end{bmatrix} \begin{bmatrix} I_Q \\ I_D \\ I_O \\ I_q \\ I_d \\ I_o \end{bmatrix} \quad (28)$$

From equations (20), (21) and (28), it is very evident that transformation of the voltage and flux linkage equations of the TFM into arbitrary qdo reference frame, has produced another set of voltage and flux linkage equations with inductance values, no longer dependent on the rotor angular position which varies with time as seen in equation (16).

IV. PHYSICAL CONFIGURATION OF AN INDUCTION MACHINE (IM)

The induction motor comprises a stator and a rotor mounted on bearings and separated from the stator by air-gap. The stator consists of a magnetic core made up of laminations carrying slot-embedded conductors which constitute the stator windings. The rotor of induction motor is cylindrical and carries either conducting bars short-circuited at both ends by end rings (squirrel cage rotor) or a polyphase winding connected in a predetermined manner with terminals brought out of slip rings for external connections and short circuited. The winding arrangement of a typical 2-pole, 3-phase, star-connected, symmetrical induction machine is as shown in figure 2.



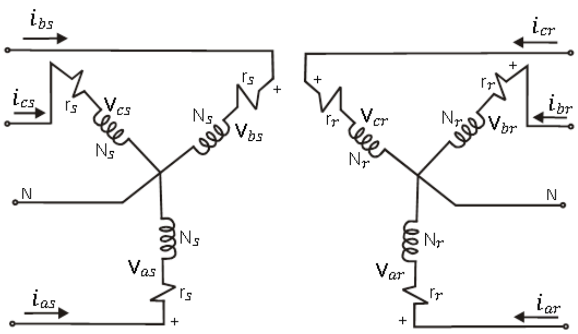


Fig.2: Two-pole, 3-phase, star-connected, symmetrical induction machine.

V. INDUCTANCE MATRIX OF AN INDUCTION MACHINE(IM)

5.1 IM Self - and Mutual - inductances.

The winding arrangement of a 2 – pole, 3-phase, star-connected symmetrical induction machine is as shown in figure (2). The stator windings are identical with equivalent turns, N_s and resistance, r_s. The rotor windings which may be wound or forged as a squirrel cage winding can also be approximated as identical windings with equivalent turns, N_r and resistance r_r. The air gap of an induction machine is uniform and the stator and rotor windings may be approximated as having a sinusoidally distributed windings.

The stator inductance, L_s, is given as;

$$L_s = \begin{bmatrix} L_{Ls} + L_A - L_B \cos 2\theta_r & -\frac{1}{2} L_A - L_B \cos 2\left(\theta_r - \frac{\pi}{3}\right) & -\frac{1}{2} L_A - L_B \cos 2\left(\theta_r + \frac{\pi}{3}\right) \\ -\frac{1}{2} L_A - L_B \cos 2\left(\theta_r - \frac{\pi}{3}\right) & L_{Ls} + L_A - L_B \cos 2\left(\theta_r - \frac{\pi}{3}\right) & -\frac{1}{2} L_A - L_B \cos 2(\theta_r + \pi) \\ -\frac{1}{2} L_A - L_B \cos 2\left(\theta_r + \frac{\pi}{3}\right) & -\frac{1}{2} L_A - L_B \cos 2(\theta_r + \pi) & L_{Ls} + L_A - L_B \cos 2\left(\theta_r + \frac{\pi}{3}\right) \end{bmatrix}$$

where;

Las as = LLs + LA – LB Cos 2qr (30)

Lbs bs = LLs + LA – LB Cos2. (θ_r-π/3) (31)

Lcs cs = LLs + LA – LB Cos2. (θ_r+π/3) (32)

Las bs = -½LA – LB Cos2 (33)

Las cs = -½LA – LB Cos2. (34)

Lbs cs =- ½LA – LB Cos2. (35)

From equation (29), it is very evident that all stator self-inductances are equal (that is;

Lasas = Lbsbs = Lcscs with; Lasas = LLs + Lms (36)

Where;

L_{Ls} = stator leakage inductance

L_{ms} = stator magnetizing inductance

The stator magnetizing inductance, L_{ms}, corresponds to L_A in equation (30) through equation (32) and is mathematically expressed as;

$$L_{ms} = .\left(\frac{N_s}{2}\right)^2 \frac{\pi \mu_o r l}{g} \quad (37)$$

Where;

N_s = stator equivalent turns

μ_o = permeability of free space

r = stator resistance

L = stator winding length

g = length of uniform air gap

Like the stator self-inductances, the stator-to-stator mutual inductances are also equal. This implies that;

$L_{asbs} = L_{ascs} = L_{bscs} = -{}^1/_2 L_{ms}$ (38)

and this corresponds to $-{}^1/_2 L_A$ in equation (33) through equation (35) with L_B = o. consequently, equation (29) is now rewritten as;

$$L_s = \begin{bmatrix} L_{Ls} + L_{ms} & -\frac{1}{2} L_{ms} & -\frac{1}{2} L_{ms} \\ -\frac{1}{2} Lms & L_{Ls} + L_{ms} & -\frac{1}{2} L_{ms} \\ -\frac{1}{2} Lms & -\frac{1}{2} L_{ms} & L_{Ls} + L_{ms} \end{bmatrix} \quad (39)$$

In a similar manner, the rotor inductance matrix is obtained as;

$$L_r = \begin{bmatrix} L_{Lr} + L_{mr} & -\frac{1}{2} L_{mr} & -\frac{1}{2} L_{mr} \\ -\frac{1}{2} Lmr & L_{Lr} + L_{mr} & -\frac{1}{2} L_{mr} \\ -\frac{1}{2} ms & -\frac{1}{2} L_{mr} & L_{Lr} + L_{mr} \end{bmatrix}$$

(40)

Whereas in stator, the rotor self-inductances are equal, that is;

Larar = Lbrbr = Lcrcr = LLr + Lmr (41)

The rotor magnetizing inductance, L_{mr}, is given as;

$$L_{mr} = \left(\frac{N_r}{2}\right)^2 \frac{\pi \mu_o r l}{g} \quad (42)$$

The rotor-to-rotor mutual inductances are equal and expressed as;

Larbr = Larcr = Lbrcr = -1/2 Lmr (43)

The mutual inductances between the stator and the rotor windings are obtained as follows;

(i) The mutual inductances L_{asar}, L_{bsbr} and L_{cscr} are equal; and is given by the expression;

Lasar = Lbsbr = Lcscr = Lsrcosqr (44)

(ii) The mutual inductances L_{asbr}, L_{bscr} and L_{csar} are equal; and is given by the expression;

$$L_{asbr} = L_{bscr} = L_{csar} = L_{sr}\cos\left(\theta_r + 2\pi/3\right) \quad (45)$$

(iii) The mutual inductances L_{ascr} L_{bsar} and L_{csbr} are equal; and is given by the expression;

$$L_{ascr}=L_{bsar}=L_{csbr}=L_{sr}\cos\left(\theta_r - 2\pi/3\right) \quad (46)$$

Equation (44) through equation (46), gives one expression for the mutual inductance between the stator and the rotor windings of an induction machine expressed as;

$$L_{sr} = L_{sr}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\theta_r \end{bmatrix} \quad (47)$$

The L_{sr} on the right-hand side of equation (47) represents the amplitude of the mutual inductances between the stator and rotor windings and is given by the expression;

$$L_{sr} = \left(\frac{N_s}{2}\right)\left(\frac{N_r}{2}\right)\frac{\pi\mu_o rl}{g} \quad (48)$$

5.2 Transformation of IM state variables to arbitrary qdo reference frame.

The voltage equations in machine variables for the stator and the rotor of a star – connected symmetrical IM shown in figure 2 are expressed as follows;

$$\left.\begin{aligned} V_{as} &= i_{as}r_s + P\lambda_{as} \\ V_{bs} &= i_{bs}r_s + P\lambda_{bs} \\ V_{cs} &= i_{cs}r_s + P\lambda_{cs} \end{aligned}\right\} \quad (49)$$

Rotor voltage equations:

$$\left.\begin{aligned} V_{ar} &= i_{ar}r_r + P\lambda_{ar} \\ V_{br} &= i_{br}r_r + P\lambda_{br} \\ V_{cr} &= i_{cr}r_r + P\lambda_{cr} \end{aligned}\right\} \quad (50)$$

In both equations, $P = {}^{d}/_{dt}$, the S subscripts denotes variables and parameters associated with the stator circuits and the r subscripts denotes variables and parameters associated with the rotor circuits. Both r_s and r_r are diagonal matrices each with equal non zero elements [1].

For a magnetically linear system, the flux linkages can be expressed as;

$$\begin{bmatrix} \lambda_s^{abc} \\ \lambda_r^{abc} \end{bmatrix} = \begin{bmatrix} L_{ss}^{abc} & L_{sr}^{abc} \\ L_{rs}^{zbc} & L_{rr}^{abc} \end{bmatrix}\begin{bmatrix} i_s^{abc} \\ i_r^{abc} \end{bmatrix} \text{wb.turn.} \quad (51)$$

For an idealized inductance machine, six first order differential equations are used to describe the machine, one differential equation for each machine winding. The stator-to-rotor coupling terms are functions of rotor position and hence when the rotor rotates, the coupling terms vary with time [5].

In the analysis of IM, it is also desirable to transform the abc variables with the symmetrical rotor windings to the arbitrary qdo reference frame [1].

And the transformation equation from the abc quantities to the qdo reference frame is given by;

$$\begin{bmatrix} f_q \\ f_d \\ f_o \end{bmatrix} = \left[T_{qdo}(\theta)\right]\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \quad (52)$$

Where the variable, f, can be the phase voltages, currents or flux linkages of the machine.

$$\left[T_{qdo}(\theta)\right] = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (53)$$

and inverse of equation (53) is;

$$\left[T_{qdo}\right]^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \quad (54)$$

5.3 IM Voltage equations in qdo reference frame

From equation (49), the stator winding abc voltage equations can be expressed as;

$$V_s^{abc} = r_s^{abc} i_s^{abc} + P\lambda_s^{abc} \quad (55)$$

where; P = d/dt

Applying the transformation, $\left[T_{qdo}(\theta)\right]$, to equation (55), yields;

$$V_s^{qdo} = \left[T_{qdo}(\theta)\right] r_s^{abc} \left[T_{qdo}(\theta)\right]^{-1} \left[i_s^{qdo}\right] + \left[T_{qdo}(\theta)\right] P \left[T_{qdo}(\theta)\right]^{-1} \left[\lambda_s^{qdo}\right] \quad (56)$$

Equation (56) can be simplified to;

$$V_s^{qdo} = r_s^{qdo} i_s^{qdo} + P\lambda_s^{qdo} + \omega \begin{bmatrix} o & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \lambda_s^{qdo} \quad (57)$$

where;

$$r_s^{qdo} = r_s \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; P = \frac{d}{dt}; \omega = \frac{d\theta}{dt}$$

In a similar manner, the rotor quantities must be transformed into the same qdo frame. Now the transformation angle for the rotor phase quantities is $(\theta - \theta_r)$. And so when the transformation, $T_{qdo}(\theta - \theta_r)$, is applied to the rotor voltage equation in the same manner as the stator, we have;

$$V_r^{qdo} = r_r^{qdo} i_r^{qdo} + P\lambda_r^{qdo} + (\omega - \omega_r) \begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \lambda_r^{qdo} \quad (58)$$

5.4 IM Flux linkage equation in arbitrary qdo reference frame

From equation (51), the stator and rotor flux linkages are given as;

$$\lambda_s^{abc} = L_{ss}^{abc} i_s^{abc} + L_{sr}^{abc} i_r^{abc} \quad (59)$$

$$\lambda_r^{abc} = L_{rs}^{abc} i_s^{abc} + L_{rr}^{abc} i_r^{abc} \quad (60)$$

The stator flux linkages in arbitrary qdo reference form are obtained by applying $T_{qdo}(\theta)$ to equation (59) to give;

$$\lambda_s^{qdo} = \left[T_{qdo}(\theta)\right]\left[L_{ss}^{abc} i_s^{abc} + L_{sr}^{abc} i_r^{abc}\right]$$
$$= T_{qdo}(\theta) L_{ss}^{abc} T_{qdo}^{-1}(\theta) i_s^{qdo} + T_{qdo}(\theta) L_{sr}^{abc} T_{qdo}^{-1}(\theta) i_r^{qdo} \quad (61)$$

Equation (61) simplifies to;

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{os} \end{bmatrix} = \begin{bmatrix} L_{ls} + \frac{3}{2}L_{ss} & 0 & 0 \\ 0 & L_{ls} + \frac{3}{2}L_{ss} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{os} \end{bmatrix} + \begin{bmatrix} \frac{3}{2}L_{sr} & 0 & 0 \\ 0 & \frac{3}{2}L_{sr} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{qr} \\ i_{dr} \\ i_{or} \end{bmatrix} \quad (62)$$

In a similar manner, if the transformation, $T_{qdo}(\theta - \theta_r)$ is applied to equation (60), the rotor qdo flux linkage becomes;

$$\lambda_r^{qdo} = \left[T_{qdo}(\theta - \theta_r)\right] L_{rs}^{abc} \left[T_{qdo}(\theta - \theta_r)\right]^{-1} i_s^{qdo} + \left[T_{qdo}(\theta - \theta_r)\right] L_{rr}^{abc} \left[T_{qdo}(\theta - \theta_r)\right]^{-1} i_r^{qdo} \quad (63)$$

Equation (63) simplifies to;

$$\begin{bmatrix} \lambda_{qr} \\ \lambda_{dr} \\ \lambda_{or} \end{bmatrix} = \begin{bmatrix} \frac{3}{2}L_{sr} & 0 & 0 \\ 0 & \frac{3}{2}L_{sr} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{os} \end{bmatrix} + \begin{bmatrix} L_{lr} + \frac{3}{2}L_{rr} & 0 & 0 \\ 0 & L_{lr} + \frac{3}{2}L_{rr} & 0 \\ 0 & 0 & L_{lr} \end{bmatrix} \begin{bmatrix} i_{qr} \\ i_{dr} \\ i_{or} \end{bmatrix} \quad (64)$$

Merging equations (62) and (64), gives the stator and rotor flux linkage equations in qdo reference frame as depicted in equation (65).

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{os} \\ \lambda_{qr}^1 \\ \lambda_{dr}^1 \\ \lambda_{or}^1 \end{bmatrix} = \begin{bmatrix} L_{ls}+L_m & 0 & 0 & L_m & 0 & 0 \\ 0 & L_{is}+L_m & 0 & 0 & L_m & 0 \\ 0 & 0 & L_{is} & 0 & 0 & 0 \\ L_m & 0 & 0 & L_{lr}^1+L_m & 0 & 0 \\ 0 & L_m & 0 & 0 & L_{lr}^1+L_m & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{lr}^1 \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{os} \\ i_{qr}^1 \\ i_{dr}^1 \\ i_{or}^1 \end{bmatrix}$$

(65)

In equation (65), the primed quantities are rotor values referred to the stator side and are related thus;

$$\lambda_{qr}^1 = \frac{N_s}{N_r}\lambda_{qr}; \quad \lambda_{dr}^1 = \frac{N_s}{N_r}\lambda_{dr} \qquad (66)$$

$$i_{qr}^1 = \frac{N_r}{N_s} i_{qr}; \quad i_{dr}^1 = \frac{N_r}{N_s} i_{dr} \qquad (67)$$

Also, from equation (65), L_m is the magnetizing inductance on the stator side and has the expression;

$$L_m = \frac{3}{2}L_{ss} = \frac{3}{2}\frac{N_r}{N_s}L_{sr} = \frac{3}{2}\frac{N_s}{N_r}L_{rr} \qquad (68)$$

CONCLUSION

From the comparative analysis carried out, it is very evident that the self-inductance matrix of the two machines, are completely independent of rotor angular position. For the TFM, this is seen in equation (13) while for the IM, it is seen in equations (39) and (40). However, the mutual coupling inductance in both cases, are dependent on rotor angular position. For the TFM in addition to rotor angle dependence, it also depends on the difference between the direct - and quadrature - axes reactances. This is as depicted in equation (16) for the TFM and for the IM, in equation (47). The comparative analysis also showed that when state variables namely voltage and flux linkage equations in both machines are transformed to arbitrary qdo reference frame, new set of voltage and flux linkage equations are obtained and whose inductance values no longer depended on the rotor angular position that varies with time; and this of course is a very big advantage in the analysis of both machines. This is as shown in equations (20), (21) and (28) for the TFM while for the IM, it is as shown in equations (57), (58) and (65).

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