On the Solution of Heat and Mass Transfer Analysis in Nanofluid Flow Using Semi-Analytic Method (HPM) Between Two Parallel Manifolds

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Abstract- This present article investigates the heat and mass transfer analysis of squeezing flow between two parallel plates. The introduction of similarity transformation on the continuity ans momentum equations under the underlying squeeze flow helped us obtain a nonlinear ordinary differential equation and then we employed a numerical scheme called finite difference method (FDM) and semi-analytic method known as Homotopy perturbation method (HPM). Taking previous work into consideration, our present work tentatively agrees and are in accordance.

Indexed Terms- Squeeze flow, Nanofluid, Magnetic field, Homotopy perturbation method (HPM), parallel manifolds, Finite Difference Method (FDM).

I. INTRODUCTION

Heat transfer in rapidly moving engines and machines with lubricant inside has been an active field of research. The squeezing flow of Newtonian and non-Newtonian fluid continue to attract significant interest of many scientific scholars due to increasing applications in diverse fields of engineering and Technology. For example, in loading of mechanical components or moving of pistons, it involves the flow between disks which allow safe and consistent working of such machine. This called for the study of heat and mass transfer in such systems.

Stefan [1] considered the squeezing flow using lubrication method 1874. In 1886, Reynolds [2] considered the problems for elliptic plates. The analysis of squeezing flow through refrigerator plates was investigated by Archibald [3] in 1956. In fact, recent efforts of researchers have made squeezing flow an interesting area of research and reliable results are been obtained. The two-dimensional axisymmetric squeezing flows between parallel plates was discussed by Rashidi et al [4]. The effects of magnetic field in squeezing flow between infinite parallel plates were reported by Sidiqui et al [5]. Squeezing flow of dusty fluid between parallel plates with the squeezing effect on the velocity profile was discussed by Hamdan and Baron [6]. The Approximate Analytic solution for the squeezing fliw of viscous fluid between disks with suction or blowing was investigated by Domaltry and Aziz [7]. Hayat et al [8] extended the work of [7], they analysed the squeezing flow of non-newtonian fluids by considering second grade fluids.

Nowadays, the study of heat and mass transfer analysis of squeezing flow of fluid has gained many attentions because of its various applications in many branches of science, engineering and Technology.

Mahmood et al [9] considered the squeezing flow of fluid through a porous surface, his results reveal that the magnitude of local Nusselt number increases with prandtl number. His results are in excellent agreement with Mustafa et al [10]. Duwairi et al [11] considered the heat transfer characteristics in a squeezing flow between parallel disks. The Analysis of Hydromagnetic effects on flow and heat transfer over a horizontal surface placed in an externally squeezed free stream were considered by Khaled and Vafai [12]. it was found that Nusselt number and wall shear stress is both increasing functions of the magnetic parameter. The analytic investigation of Unsteady squeezing flow of viscous jeffery fluid between parallel disks was considered by Qayyum et al [13]. porosity and squeezing effects on the velocity profiles was investigated.

Advancement in technology requires further improvement in this Area of research, especially the heat transfer from energy saving point of view. Modern science bestowed blessing in the form of nanofluid in which nano-sized particles are added to the basic fluid to increase the heat transfer capabilities of the base fluid. Note that there have been many research articles recently published [13,18] on the mathematical and numerical, and modelling of convective heat transfer mechanism in nanofluids. Mixed convection flow and heat transfer of nanofluids due to an unsteady stretching sheet was considered by Mandy [19]. These models have some advantages over experimental studies due to several factors that influences nanofluid properties.

Hatami et al [20,21] Recently considered nanofluid flow and heat transfer characteristics under different flow configuration. Pourmehran et al [22] investigated the Analysis of Unsteady squeezing nanofluid flow, it was shown from his result that the highest value of Nusselt number can be obtained by selecting silver as nanoparticle. Infact, it was deduced that All the above-mentioned models are single phase model. A new dimension has been added by [23,24] in terms of complex geometry or two-phase model for simulating nanofluid.

Science and Engineering fields are endowed with so many numerical and analytical methods to achieve or obtain accurate approximate solutions from non-linear equations. These methods are Variation of parameter (VPM), [25,26], differential transformation method (DTM) [27,31], Adomian decomposition method (ADM), [32,33], Homotopy analysis method (HAM), [34,35], Homotopy perturbation method (HPM),[36], etc. In this article we applied relatively a semi analytic technique say Homotopy perturbation method (HPM). This technique is presented in the numerical experiment.

Motivated by the above investigations, the present article deals judiciously with the squeeze flow of fluid between two parallel manifolds. The introduction of similarity transformation has been used to obtain non-linear ordinary differential equation from underlying governing equations. The resulting non-linear differential equation has been solved with the help of Homotopy Perturbation Method (HPM) and Numerically by Finite Difference Method (FDM).

II. MATHEMATICAL FORMULATION

2.1 Flow Analysis

We put the viscous incompressible Nanofluid flow into consideration and also the transfer of heat in the coordinate system. The x-axis in tyhe coordinate system chosen is measured along the disk(manifold) and the y-axis is perpendicular (Normal) to the disk(manifold). The squeezing flow is performed through a system having parallel manifolds situated opposite to each other. $h(t) = H\,(1-t)^{\frac{1}{2}}$ distance apart where > 0 refers to the squeezing movement of the both disks with velocity $v(t) = \frac{dh}{dt}$ until they touch each other at $t = \frac{\grave{}}{\alpha}$, Also, < 0 refers to movements of the disk away to each other and is called the characteristics parameter having dimension of time inverse. H is the initial position of the plate at time $t = 0$ A uniform magnetic field of strength $b(t) = B_0(1-t)^{-\frac{1}{2}}$ is applied normal to the disk where B_0 is the initial intensity of the magnetic field ,In the mathematical formulation scheme we proceed with the following assumption that there is no chemical reaction, radiative heat transfer of nano-particles and base fluid are in thermal equilibrium and nonstop occurs between them. All body force is assumed to be neglected. Here we have considered Nano fluids. The thermos physical properties of the nanofluid are given in

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{1}{\mu_{nf}}\frac{\partial \rho}{\partial x} + \\ + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma\beta_o^2(t)u}{\mu_{nf}} \tag{2}$$

$$\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{1}{\mu_{nf}}\frac{\partial \rho}{\partial y} + \\ + \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{\mu_{nf}}{(\rho C_p)_{nf}}\left[4\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2\right] \tag{4}$$

Where u, v are the velocity components in x and y directions respectively. Here T, μ_{nf}, $(\rho C_p)_{nf}$, K, p, ρ_{nf} represents the Temperature, effective dynamic viscosity, effective heat capacity, effective thermal conductivity and effective Density of the nanofluids respectively. Now the relations between base fluid and nano particles are given by

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s \tag{5}$$

$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s \tag{6}$$

$$\mu_{nf}=\frac{\mu_{nf}}{(1-\phi)^{2.5}}(Brinkman) \tag{7}$$

$$\frac{K_{nf}}{K_f}=\frac{K_s+2K_f-2\phi(K_f-K_s)}{K_s+2K_f+\phi(K_f-K_s)}\ (maxwell-Garnelts)$$
$$\sigma_{nf}=(1-\phi)\sigma_f+\phi\sigma_s \tag{8}$$

The relevant boundary conditions for the present problem are as follows:

$$u=0, v=\frac{dh}{dt}, T=T_H \quad at \quad y=h(t)$$
$$u=0, v=0, \frac{dT}{dh}=0 \quad at \quad y=0 \tag{9}$$

Now in order to convert the partial differential equation into ordinary differential equations. let us introduce the dimensionless function f, θ and similarly variable $\sim$ as follows.

$$u=\frac{\alpha x}{2(1-\alpha t)}f'(\sim) \quad , \quad \theta=\frac{T}{T_h}$$
$$v=\frac{-\alpha H}{(1-\alpha t)}f(\sim) \quad ,$$
$$B(t)=B_o(1-\alpha t)^{-\frac{1}{2}} \quad ,$$
$$\sim=\frac{y}{H(1-\alpha t)^{-\frac{1}{2}}} \tag{10}$$

where primes denote the differentiation with respect to eta. Using the above transformation and simplifying Equations (2), (3) by eliminating pressure terms, we have

$$f^{iv}-S(\sim f'''+3f''+f'f''-f'f''')-M^2f''=0 \tag{11}$$

and by using the above transformation and simplifying Equation (4) yield

$$(1+\mu_r)\theta''+P_rS(f\theta'-\sim\theta')+P_rE_c(f''^2+4\delta^2f'^2)=0 \tag{12}$$

with the association of the boundary conditions.

$$f''(0)=0, f(0)=0, \theta'(0)=0$$
$$\phi'(0)=1$$
$$for \quad \sim=0$$
$$f'(1)=0, f(1)=1, \theta'(1)=1$$
$$\phi(1)=0$$
$$for \quad \sim=1 \tag{13}$$

where $A_1=(1-\phi)+\phi\frac{\mu_s}{\mu_f}$,

$A_2=(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}$, $A_3=k_s+2k$ and $S=\frac{\alpha H^2}{2\sqrt{_f}}$ denotes the squeeze number having the property $s>0$ corresponds to the disks moving apart and $s<0$ corresponds to the disks moving to each other.

$P_r=\frac{\mu_f(\rho C_p)_f}{\mu_f k_f}$ is the prandtl number, $E_c=\frac{\mu_f}{(\rho C_p)_f T_H}\left[\frac{\alpha x}{2(1-\alpha t)}\right]^2$ is the modified Eckert number, $M=\frac{\sigma_{nf}}{\mu_{nf}}H^2\alpha B_o^2$ is the Hartman number, and $\delta=\frac{H(1-\alpha t)}{\alpha}$ is the dimensionless Length.

2.2 physical quantities

For practical purpose the skin friction coefficient and Nusselt number can be defined respectively as follows:

$$C_f = \frac{\mu_{nf}(\frac{\partial u}{\partial y})_{y=h(t)}}{\frac{1}{2}\rho_{nf}(\frac{dh}{dt})^2}$$

$$Nu = \frac{-HK_{nf}(\frac{\partial T}{\partial y})_{y=h(t)}}{K_f T_H} \qquad (14)$$

In terms of dimensionless variables, the reduced skin friction coefficient and the reduced Nusselt number are respectively given by

$$C_{fr} = \frac{Re_s C_f H^2 \sqrt{1-\alpha t}}{2\alpha^2} = \frac{f''(1)}{A_1(1-\phi)^{2.5}}$$

$$Nu_r = Nu \cdot \sqrt{1-\alpha t} = -A_3\theta'(1) \qquad (15)$$

III. NUMERICAL EXPERIMENT

3.1 Homotopy perturbation method

In this section we illustrate the basic idea of the HPM. For this we consider the following differential equation:

$$A(u) - f(r) = 0, r\epsilon\Omega \qquad (16)$$

$$B\left(u, \frac{\partial u}{\partial r}\right) = 0, r\epsilon\Gamma \qquad (17)$$

where A represents a general differential operator, B is a boundary operator, ˆI is the boundary of the domain ˆI c , and $f(r)$ is known analytic function. The operator A can be decomposed into two parts viz. linear (L) and nonlinear (N). Therefore, Eq. (17) may be written in the following form:

$$L(u) + N(u)af\hat{}\,(r) = 0 \qquad (18)$$

An artificial parameter p can be embedded in Eq. (19) as follows:

$$L(u) + p(N(u)af\hat{}\,(r)) = 0, \qquad (19)$$

where p [0,1]is the embedding parameter (also called as an artificial parameter).

Using homotopy technique, we construct a homotopy $v(r, p)$:
$\Omega \times [0,1] \rightarrow R$ to Eq. (19) which satisfies

$$H(v,p) = (1\hat{a}p)[L(v)\hat{a}L(u_0)] + p[L(v) + N(v)\hat{a}f(r)] = 0 \qquad (20)$$

$$H(v,p) = L(v)\hat{a}L(u_0) + pL(u_0) + p[N(v)\hat{a}f(r)] = 0. \qquad (21)$$

Here, u_0 is an initial approximation of Eq. (22) which satisfies the given conditions.

By substituting $p = 0$ and $p = 1$ in Eq. (22), we may get the following equations, respectively.

$$H(v,0) = L(v)aL\hat{}\,(u_0) \qquad (22)$$

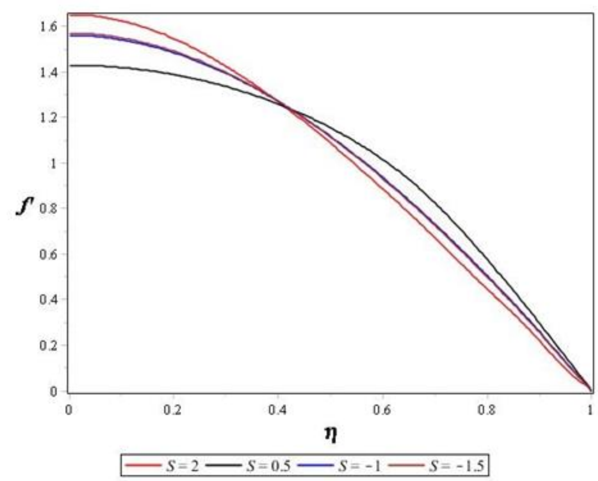

Figure 1: Effects of squeezing parameter s on velocity

and

$$H(v,1) = A(v)af\hat{}\,(r) = 0. \qquad (23)$$

As p changes from zero to unity, $v(r, p)$ changes from $u_0(r)$ to $u(r)$. In topology, this is called deformation and $L(v)aL\hat{}\,(u_0)$ and $A(v)af\hat{}\,(r)$ are homotopic to each other. Due to the fact that p[0,1] is a small parameter, we consider the solution of Eq. (21) as a power series in p as below

$v = v_0 + pv_1 + p^2v_2 + \cdots$ (24)

The approximate solution of Eq. (17) may then be obtained as

$u = \lim v = v_0 + v_1 + v_2 + \cdots$ (25) $p \to 1$

3.2 Code of verification

To check the efficiency of FDM, a comparison between semianalytic solution obtained by HPM and numerical solution obtained by FDM is tabulated in Table 1, Here we have calculated the values of $f^{00}(1)$ of the nanofluid for various values of squeeze number S. The results obtained are in good agreement.

Table 1: The values $f^{00}(1)$ for various values of S

S	$f^{00}(1)$	
	HPM	FDM
0.2	-3.639586	-3.639569
0	-3.194551	-3.194528
-1.0	-2.719280	-2.719288
-1.5	-2.650898	-2.650888
-2.0	-2.608568	-2.608566

We have calculated the values of Nusselt $-\theta^0(1)$ for various values of P_r and E_c and tabulated these values in Table 2. We see that the values are in excellent agreement with Mustapha et al. [10] and Pourmehran et al. [22].

Table 2: The values of $\theta^0(1)$ for various values of P_r, E_c

P_r	E_c	Mustafa et al	Pourmehran et al	Present work
		[9]	[22]	
0.5	1.0	1.522368	1.518859607	1.527258539
1.0	-	3.026324	3.019545607	3.025481014
2.0	-	5.98053	5.967887511	5.950384834
5.0	-	14.43941	14.41394678	14.421072431
1.0	0.5	1.513162	1.509772834	1.506085793
-	1.2	3.631588	3.623454726	3.623586179
-	2.0	6.052647	6.039091204	6.058376658

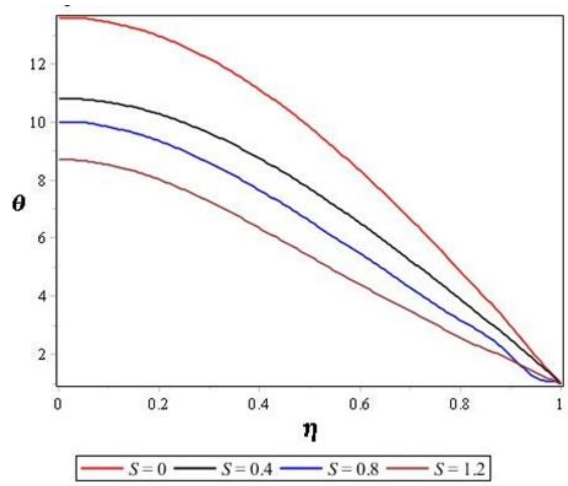

Figure 2: Effect of squeezing parameters on temperature

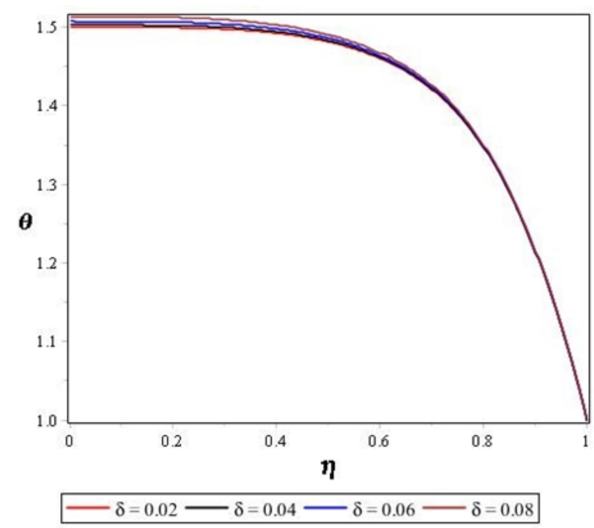

Figure 3: Effects of nanoparticle volume fraction on temperature when $S > 0$

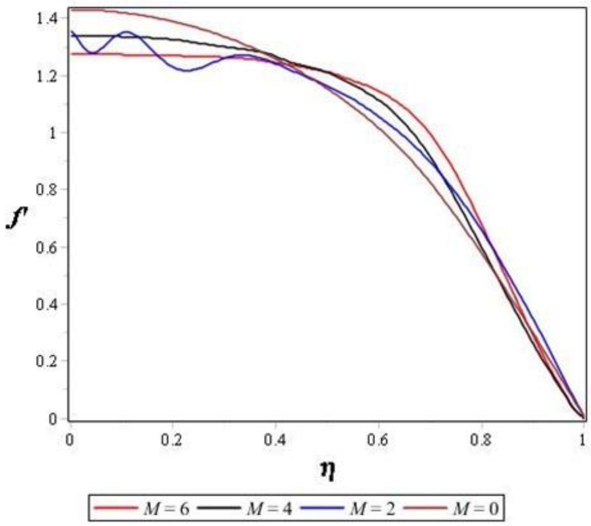

Figure 4: Effects of Hartmann number M on temperature when $S > 0$

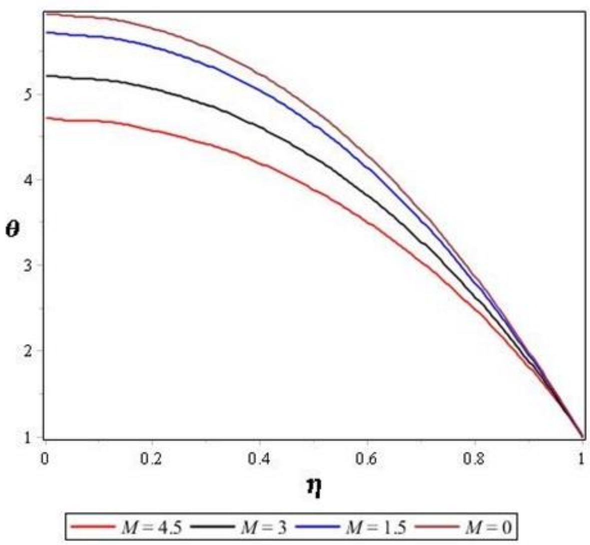

Figure 5: Effects of Hartmann number M on temperature when $S < 0$

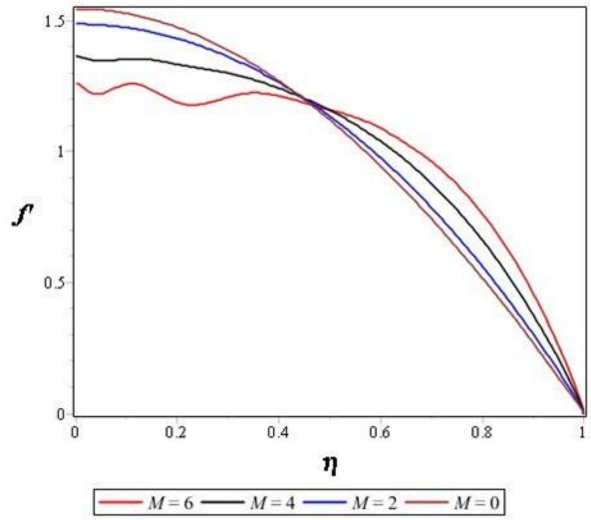

Figure 6: Effects of Hartmann number M on velocity when $S < 0$

IV. RESULT AND DISCUSSION

In this section we will discuss the effects of various physical parameters on velocity distribution. The whole discussion has been performed through graphs and tables. In the simulation the default values of parameters are taken as $P_r = 1.0$, $E_c = 0.5$, $M = -1.0$, $S = -0.5$, $\delta = 0.1$, $\gamma = 0.5$

4.1 Effect of S on velocity and temperature profiles

fig (1) demonstrate the influence of the squeezing parameter S on the fluid velocity. It reveals that for 0 ≤~≤ 0.42 (not accurately determined) the velocity profile decreases with increasing S but for ~≥ 0.42 (not accurately determined) it starts increasing as shown in the figure.

Physically this can be explained that the decrease in the fluid velocity near the wall region causes to increase velocity gradient there. since the mass flow rate is kept constant, hence decrease in the fluid velocity near the region will be compensated by the increasing fluid velocity near the central region. That is why at ~= 0.42 (not accurately determined) we found a point of separation and backflow occurs. Also, the rate of decrease in the fluid.

4.2 Effect of S on velocity and temperature profiles

The impact of Hartmann number M on velocity distribution has been portrayed in fig (4) and fig (5) for both $S < 0$ and $S > 0$. It reveals that as M increases the velocity decreases for 0 ≤∼≤ 0.42 (not accurately determined) and it is true because for electrically conducting fluid in the presence of magnetic fluid there will always be Lorentz force which slows the motion of fluid in the boundary layer region. But for ∼> 0.42 (not accurately determined) the reverse effect is seen because of the same mass flow reason as discussed in the previous section (4.1). Moreover, when $S < 0$ two disks or plates or manifolds are very close to each other, then the situation together with retarding Lorentz force creates adverse pressure gradient. Whenever such forces act over a long time then there might be a point of separation and backflow occurs. For $S > 0$ the reason is slightly different. When two plates move apart then a vacant space occurs and fluid in that region goes with high velocity so that mass flow conservation will not be violated. That is why for ∼> 0.42 we suddenly find an accelerated flow.

Fig (4) and fig (5) exhibits that the temperature of the fluid flow decreases as Hartmann Number M increases for both $S < 0$ and $S > 0$.

V. CONCLUSION

In the present article the influence of magnetic field on squeezing flow of nanofluids has been discussed. The model is transformed and rendered into dimensionless form and then solved semi-analytically using Homotopy Perturbation Method (HPM) as well as numerically using FDM together with comparison to show the efficiency of HPM. The numerical discussion has been performed through graphs and tables to illustrate the details of the flow characteristics. Based on the whole discussion the main conclusions of our investigations are as follows. It is observed that fluid velocity decreases with the influence of Squeezing parameter and Hartmann number within the region 0 ≤∼≤ 0.42.

REFERENCES

[1] M.J. Stefan, Versuchs Uber die scheinbare adhesion, Sitzungsberichte der Akademie der Wissenschaften in Wien, MathematikNaturwissen, 69 (1874) 713-721.

[2] O.Reynolds, On the theory of lubrication and its application to Mr. Beauchamp Tower's experiments including an experimental determination of the viscosity of olive oil, Philos Trans. R. Soc. Lond. 177(1886) 157-234

[3] F.R. Archibald. Load capacity and time relations for squeeze films. J. Lubr. Technol

[4] M.M Rashidi, H. Shahmohamadi, S. Dinarvand. Analytic approximate solutions for unsteady two-dimensional and axisymmetric squeezing flows between parallel plates. Math Probl. Eng (2008) 1-13.

[5] A.M Siddiqui, S.Irum, A.R. Ansari. Unsteady squeezing flow of a viscous MHD fluid between parallel plates.Math. Modell. Anal. 13 (2008) 565-576.

[6] M.H. Hamdan, R.M. Baron. Analysis of the squeezing flow of dusty fluids. Appl. Sci. Res. 49 (1992) 345-354

[7] G. Domairiy, A. Aziz. Approximate analysis of MHD squeeze flow between two parallel disks with suction or injection by homotopy perturbation method, Math. Prob. Eng. (2009) 603-616

[8] T. Hayat, A. Yousaf, Mustafa, S. Obaidat, MHD squeezing flow of second grad fluid between parallel disks. Int. J. Num. Meth. Fluids 69 (2011) 399-410.

[9] M. Mahmood, S. Asghar, M.A. Hossain, Squeezed flow and heat transfer over a porous surface for viscous fluid. Heat Mass Transf. 44 (2007) 165-173.

[10] M. Mustafa, T. Hayat, S. Obaidat, On heat and mass transfer in the unsteady squeezing flow between parallel plates. Mechanics 47 (2012) 1581-1589

[11] H.M. Duwairi, B. Tashtoush, R.A. Domesh, On heat transfer effects of a viscous fluid squeezed and extruded between parallel plates, Heat Mass Transf. 14 (2004) 112-117.

[12] A.R.A Khaled, K. Vafai, Hydromagnetic squeezed flow and heatb transfer over a sensor surface, Int. J. Eng. Sci. 42 (2004) 509-519

[13] A. Qayyum, M. Awais, A. Alsaedi, T. Hayat. Squeezing flow of non-Newtonian second grade

fluids and micro-polar models, Chin. Phys. Lett 29 (2012) 034701.

[14] A. Malvandi, D.D. Ganji, Magnetic field effect on nanoparticles migration and heatbtransfer of water aluminananofluid in a channel. J. Mag. Mater. 362 (2014) 172-179

[15] M.A.A. Hamad, I. Pop, Md.A.L. Ismail, Magnetic field effects on free convection flow of a nanofluid past a vertical semi-infinite flat plate, Nonlinear Anal.. Real World Appl. 12 (2011) 1338-1348.

[16] A. Malvandi, S.A. Moshizi, D.D. Ganji, Effect of magnetic fields on heat convection inside a concentric annulus filled with Al3O3 water nanofluid, Adv. Powder Technol. 25 (2014) 18171824.

[17] A. Malvandi, D.D. Ganji, Magnetic field and slip effects on free convection inside a vertical enclosure filled with alumina water nanofluid, Chem. Eng. Res. Des. 94 (2015) 355-364.

[18] X. Wang, X. Xu, S.U.S. Choi, Thermal conductivity of nanoparticle-fluid mixture, J. Thermophys. Heat Transf. 13 (1999) 474-480

[19] A. Mandy, Unsteady mixed convection boundary layer flow and heat transfer of nanofluids due to stretching sheet, Nucl. Eng. Des. 249 (2012) 248-255.

[20] M. Hatami, M. Sheikholeslami, D.D. Ganji, Nanofluid flow and heat transfer in an asymmetric porous channel with expanding or contracting wall, J. Mole Liquids 195 (2014) 230-239.

[21] M. Hatami, M. Sheikholeslami, M. Hosseini, D.D. Ganji, Analytical investigation of MHD nanofluid flow in non-parallel walls, J. Mole Liquids 194 (2014) 251-259.

[22] O. Pourmehran, M. Rahimi-Gorji, M. Gorji-Bandpy, D.D. Ganji, Analytical investigation of squeezing unsteady nanofluid flow between parallel plates by LSM and CM. Alexandria Eng. J. 54(2015) 17-26.

[23] M. Sheikholeslami, S. Soleimani, D.D. Ganji, Effect of electric field on hydrothermal behaviour of nanofluid in a complex geometry, J. Mol. Liq. 213 (2016) 153-161.

[24] M. Sheikholeslami, M.M. Rashidi, D.D. Ganji, Numerical investigation of magnetic nanofluid forced convective heat transfer in existence of variable magnetic field using two phase model, J. Mol. Liq. 212 (2015) 117-126.

[25] M.A. Noor, S.T. Mohyud-Din, A. Waheed, Variation of parameter method for solving fifth-order boundary value problems. Appl. Math. Inform. Sci. 2 (2008) 135-141.

[26] S.T. Mohyud-Din, A. Yildirim, Variation of parameter method for fisher’s equation. Adv. Appl. Math. Mech. 2 (2010) 379388.

[27] F. Ayaz. On the two-dimensional differential transformation method. Appl. Math. Comput. 143 (2007) 361-374.

[28] M.J. Jang, C.L. Chen, Y.C. Lin, Two-dimensional differential transformation method for solving partial differential equation. Appl. Math. Comput. 121 (2001) 261-270.

[29] M. Sheikholeslami, M. Azimi, D.D. Ganji. Application of differential transformation method for nanofluid flow in a semipermeable channel considering magnetic field effect, J. Comput. Meth. Eng. Sci. Mech. 16 (2015) 246-255.

[30] M. Sheikholeslami, M.M. Rashidi, D.M. Alsaad, H.B. Rokni, Steady nanofluid flow between parallel plates considering thermophoresis and Brownian effects, J. King Saud Univ.-Sci. (2015), http://dx.doi.org/10.1016/j.ksus.2015.06.003.

[31] M. Sheikholeslami, D.D. Ganji, Nanofluid flow and hweat transfer between parallel plates considering Brownian motion using DTM, Comput. Meth. Appl. Mech. Eng. 283 (2015) 651-663.

[32] M. Sheikholeslami, D.D. Ganji, H.R. Ashorynejad. Investigation of squeezing unsteady nanofluid flow using ADM. Powder Technol, 239 (2013) 259-265.

[33] M. Sheikholeslami, D.D. Ganji, H.R. Ashorynejad, H.B. Rokni, Analytical investigation of jeffery hamel flow with high magnetic field and nanoparticle by Adomian decomposition method, Appl. Math. Mech. Engl. Ed. 33 (2012) 1553-1564.

[34] R. Ellahi, M. Raza, K. Vafai, Series solutions of non-Newt onian nanofluids with Reynolds Model and Vogel's model by means of the homotopy analysis method, Math. Comput. Modell. 55 (2012) 1876-1891

[35] M. Sheikholeslami, R. Ellahi, H.R. Ashorynejad, T. Hayat, Effects of heat transfer in flow of nanofluids over a permeable stretching wall in a porous medium. J. Comput. Theor. Nanosci, 11 (2014) 486-496.

[36] M. Sheikholeslami, D.D. Ganji, Heat transfer of Cu-water nanofluid between parallel plates, Powder Technol. 235 (2013) 873-879.

[37] J.K. Zhou, Differential Transformation and its Application for Electrical Circuits. Huazhong University Press. China, 1986.

[38] Nilankush Acharya, Kalidas Das, Prabir Kumar Kundu." The squeezing flow of Cu-water and Cu-kerosene nanofluids between two parallel plates". Alexandria Eng. J. 55 (2016) 1177-1186.

[39] Snehashish Chakraverty, Nisha Rani Mahato, Perumandla Karunakar, Tharasi Dilleswar Rao." Advanced Numerical and Semi-Analytical Methods for Differential Equations", Wiley, 2019.