

Design of Full State Feedback Controller for DC Motor Based Industrial Conveyor System

I.E. EKENGWU¹, O.G. UTU², U.D. OGBONNA³

^{1,3} Department of Mechanical Engineering, Nnamdi Azikiwe University, Awka, Nigeria

² Department of Welding and Fabrication Technology, Delta State Polytechnic, Ogwashiuku, Nigeria

Abstract- *The use of state space in the representation of plant dynamics provides knowledge not only for the input and output relationship, but also for the internal variables performance. This paper has presented design of full state feedback controller for Direct Current (DC) motor based industrial conveyor system. The dynamics of industrial conveyor system have been represented in state space form. A full state feedback control technique has been applied to the control loop of industrial conveyor system. The objective is to improve the time domain performance parameters of a conveyor system in terms of reduce lines speed error for faster movement of materials or item in synchronization with the speed of operation of other processing plant within the manufacturing arrangement. This in this case is taken to be the effectiveness of the controller to track a step input representing the desired speed of operation in a very fast manner (settling time of 0.95 s) and reduced error tracking with overshoot of 4% obtained from the simulation conducted in MATLAB environment.*

Indexed Terms- *Conveyor system, Direct Current, Full state feedback controller, Speed*

I. INTRODUCTION

Conveyor is a mechanical system that is used for material handling purpose especially in manufacturing processes and cargo operations. Conveyor systems are present in various types of industrial plants [1]. The operation of conveyor is usually automated. Industrial conveyor systems are said to be labour saving since large volumes are allowed to move quickly through a process, thereby enabling companies to ship or collect higher volumes with small storage and with a reduced amount of labour expense. These systems are usually driven by Direct Current (DC) motors. The drive signal of the motor is produced by the control component of the conveyor system to change the

speed according to the calculated deviation. This unit uses intelligent algorithm, programmable logic controller (PLC), Proportional Integral and Derivative (PID) and hybrid algorithm to regulate the motor speed

Direct Current (DC) motors are largely employed in belt drive systems including conveyor belt systems. Serving as transducers, DC motors convert electrical energy into mechanical energy operation. One of the foremost characteristics of high-performance motor drive system is usually good dynamic speed command tracking and load control response. Adding a controller makes the loop automated and more efficient. Hence, many control techniques have been proposed and implemented for aiding the working of industrial conveyor for high speed performance.

A PID control technique has been implemented for industrial conveyor system by Umoren et al [2] to address the problem arising due to the lack of synchronization of conveyor line speed and the speed of all other process machines in a brewery. Similarly, Ufot et al. [1] used Fuzzy Logic Controller (FLC) to achieve synchronized line speed control for an industrial conveyor system. Adaptive Fuzzy-PID controller has been used by Mohd [3] to control a DC motor driven belt conveyor system. Application of polynomial stabilization method for modelling and PID control of belt conveyor system has been presented by Wahyudie and Kawabe [4]. Positioning control for accurate tracking was carried out by Selezneva [5] using PID scheme applied to a linear belt-drive system. However, there is need to improve the performance of a conveyor control system in terms of reduce lines speed error for faster movement of materials or an item in synchronization with the speed of operation of other processing plant within the manufacturing arrangement.

In this paper, the dynamic of DC motor driven industrial conveyor system is being studied in terms of state space modeling, and a full state feedback control technique applied to control the system. The remaining part of this paper is divided into four sections namely, state space modeling and control design, simulation results and discussion, and conclusion.

II. STATE SPACE MODELLING AND CONTROL DESIGN

A. State Space Modelling

A system of linear differential equations can be represented in many different ways. One of these several ways is the transfer function equation, which is usually based on a simple input-output representation of plant model [6]. In spite of the fact that representing a plant in the transfer function form offers simple and powerful analysis and design method [7], the knowledge of the interior structure of the plant is being used in these method (Rowell).

Another way a plant can be expressed is the state space form. For instance, a Single Input Single Output (SISO) Linear Time Invariant (LTI) system can be represented in the state space form given by:

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx + Du \tag{2}$$

where x is an $n \times 1$ vector representing the state variables of the system, u and y are scalars representing the input and output respectively. Also, A, B, C represent the $(n \times n)$, $(n \times 1)$, and $(1 \times n)$ matrices that determine the relationships between the state variables and the input and output. While D is the input disturbance matrix. Though state space representation can be employed for Multiple Input Multiple Output (MIMO) systems, in this paper the concern will be on SISO systems.

- Conveyor System: In order to apply the state space control design control method, a DC motor based industrial conveyor system whose transfer function is given by Umoren et al (2016) is presented below:

$$G_p(s) = \frac{0.1181s - 41.74}{s^3 + 29.63s^2 + 1.217s - 2.362} \tag{3}$$

The following MATLAB m-file was used to obtain the state-space model of the system:

```
num = [0.1181 - 41.74];
den = [1 29.63 1.217 - 2.362];
Gp = tf(num, den);
[A B C D] = ssdata(Gp)
```

where

$$A = \begin{bmatrix} -29.63 & -0.6085 & 1.181 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [0 \ 0.0148 \ -5.2175]$$

$$D = 0$$

Hence the system in state space form is given by:

$$\dot{x} = \begin{bmatrix} -29.63 & -0.6085 & 1.181 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} u \tag{4}$$

$$y = [0 \ 0.0148 \ -5.2175] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{5}$$

- System Properties: There are certain properties of system that should be studied from the perspective of state-space control method. This is necessarily important considering the fact that each property will have impact on the control method. The properties are considered as follows.

a) Stability

Stability is a strategic property to understanding the dynamic characteristics of a system. Hence, it is one of the properties that should be analyzed to ascertain whether the system is stable without any control. This is achieved in this case considering the eigenvalues of the system matrix A , which is equal to the poles of the transfer function, and determines the stability. For the

system matrix A , the eigenvalues are the values s that are solutions of $\det(sI - A) = 0$. This is obtained using the MATLAB expression given by:

$$poles = eig(A)$$

poles =

-29.5862

-0.3053

0.2615

Looking at the poles, it can be seen that one of the poles is positive, which indicates that it is located at the right half plane (RHP) of the s -plane, and this means that the open-loop system is unstable. Since the system is unstable, it requires a controller.

b) Controllability

A system is said to be controllable if a control input $u(t)$, always exists that transfers any state of the system to any other state in finite time. For an LTI system, it can be said to be controllable if and only if its controllability matrix C_o has full rank. The controllability matrix and its rank are obtained as given by:

$$C_o = [B \quad AB \quad A^2B] \tag{6}$$

$$AB = \begin{bmatrix} -118.52 \\ 8 \\ 0 \end{bmatrix}, A^2B = \begin{bmatrix} 3506.9 \\ -237 \\ 8 \end{bmatrix}$$

$$C_o = \begin{bmatrix} 4 & -118.5 & 3506.9 \\ 0 & 8 & -237 \\ 0 & 0 & 8 \end{bmatrix}$$

Hence,

The rank of the matrix is obtained using the MATLAB command: $rank(C_o)$ and this gives 3. Since the controllability matrix is of order 3×3 , the system is controllable.

c) Observability

There is the possibility of having all the state variables of a system not to be directly measurable. In which case, it is essential to estimate the values of the unknown internal state variables using the available outputs. Hence, if the initial state $x(t_o)$, can be completely identified based on knowledge of the system input, $u(t)$, and system outputs, $y(t)$, over a finite time interval $t_o < t < t_f$, the system is said to be

observable. For LTI system, the observability matrix O_b must have a full rank for the system to be observable. Thus:

$$O_b = [C \quad CA \quad CA^2]^T \tag{7}$$

$$CA = [0.0295 \quad -5.2175 \quad 0]$$

$$CA^2 = [-11.3098 \quad -0.0180 \quad 0.0349]$$

$$O_b = \begin{bmatrix} 0 & 0.0148 & -5.2175 \\ 0.0295 & -5.2175 & 0 \\ -11.3098 & -0.0180 & 0.0349 \end{bmatrix}$$

Hence,

The rank of the observability matrix obtained by entering the command $rank(O_b)$ in MATLAB is 3. With the observability matrix being 3×3 , and the rank 3, the system is completely observable.

B. System Design and Configuration

A full state feedback controller is designed in this section using pole placement technique. The choice of full state feedback is due to the fact that it is more robust than other pole placement methods. The block diagram of a full state feedback system is shown in Fig. 1. The term full state implies that all state known to the controller at all times.

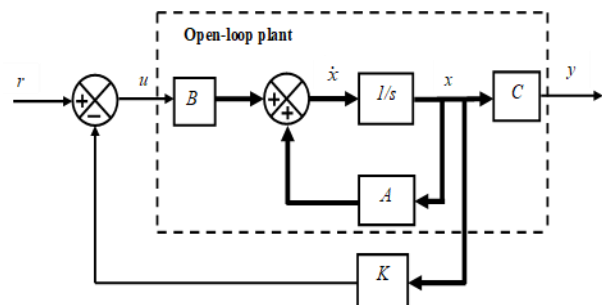


Fig. 1 Full state feedback system

In order to present the full state feedback as a pole placement design approach in which all desired poles are selected at the beginning of the design process, Fig. 1 is considered assuming the reference input r , is zero, then:

$$u = -Kx \tag{8}$$

where u is the control input, K is the feedback gain, and x is a state variable. Substituting Eq. (8) into Eq. (1) gives:

$$\dot{x} = (A - BK)x \tag{9}$$

which has a solution of $x(t) = x(0)e^{-(A-BK)t}$ [8].

Thus by proper selection of the gains of K , it can adjust the response of the system as desired.

The design objective is for the conveyor system to be able to move within 98%, that is 2% criterion, of the desired speed in less than or equal to 0.5s (representing settling time), and having overshoot of less or equal to 5% in the production process. These design specifications were chosen because they are ideal for practical application and are usually employed in control literature. Using these performance specifications, the closed-loop poles are placed with two dominant poles at $-20 \pm 20i$ with the third pole placed at -100 such that it is suitably fast and will not have much impact on the output. Hence, the values of the feedback gain K are obtained using the MATLAB command from m-file given:

$$\begin{aligned}
 p1 &= -20 + 20i \\
 p2 &= -20 - 20i \\
 p3 &= -100 \\
 K &= \text{place}(A, B, [p1 \ p2 \ p3]) \\
 &= [28 \ 600 \ 10000]
 \end{aligned}$$

There is possibility of the output not being able to track or follow the desired or reference input by using only the feedback gain, which results to expression: Kx . Therefore, to address this inadequacy of using K alone in the closed loop, a forward path gain is implemented as shown in Fig. 2.

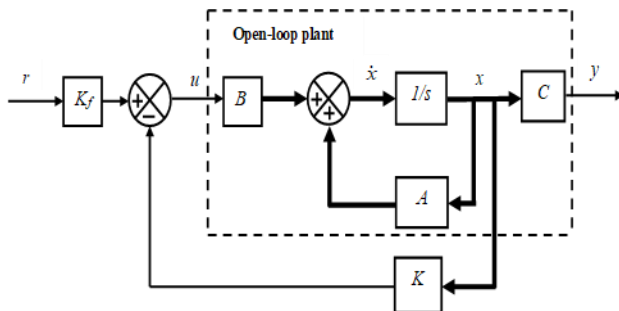


Fig. 2 Full state feedback with forward path gain

Considering Fig. 2, the control input can be given by:

$$u = K_f r - Kx \tag{10}$$

where K_f is the forward path gain, r is the reference input. Substituting Eq. (10) into Eq. (1) gives:

$$\dot{x} = (A - BK)x + BK_f r = A_{CL}x + B_{CL}r \tag{11}$$

where $A_{CL} = (A - BK)$ and $B_{CL} = BK_f$. The transfer function of the system can be given by:

$$G_{CL}(s) = \frac{Y(s)}{U(s)} = C_{CL} \phi_{CL} B_{CL} = C(sI - A_{CL})^{-1} B_{CL} K_f \tag{12}$$

Applying the final value theorem and given a step input as reference signal, then:

$$\begin{aligned}
 y(\infty) &= \lim_{s \rightarrow 0} sY(s) = sC_{CL}(sI - A_{CL})^{-1} B_{CL} K_f \frac{r}{s} \\
 &= -C_{CL}(A_{CL})^{-1} B_{CL} K_f r
 \end{aligned}
 \tag{13}$$

The value K_f is calculated within MATLAB to be $-1.9166e + 06$. Hence, the structured of the proposed system is shown in Fig. 3.

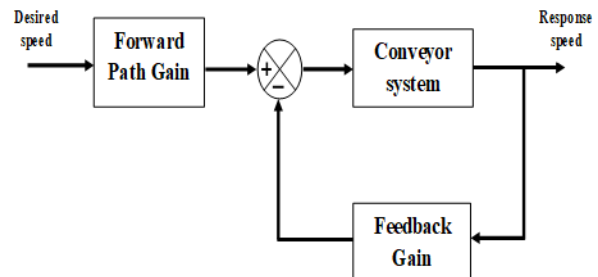


Fig. 3 Structure of proposed conveyor system

III. RESULTS AND DISCUSSION

The results obtained from the simulations conducted using developed m-file of MATLAB are shown in Fig.4 to 6. Since there are several performance indices employed in evaluating a designed system whose mechanical operation is being controlled. In this paper, the time domain indices are employed.

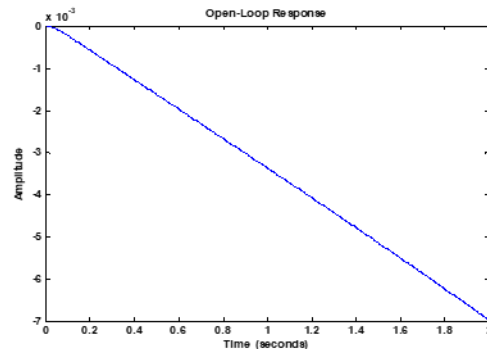


Fig. 4 Open-loop response of conveyor system

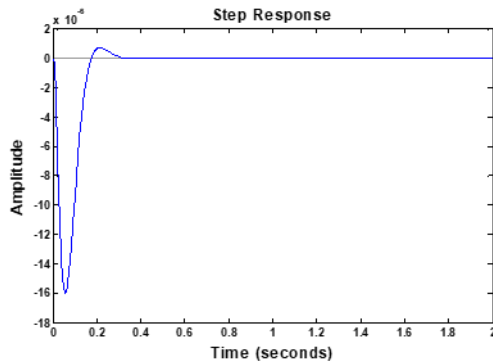


Fig. 5 Step response of conveyor system (no forward path gain)

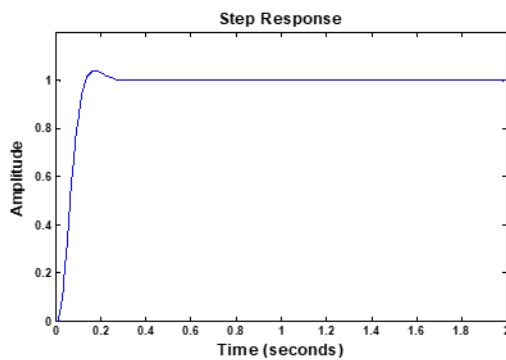


Fig. 6 Step response of conveyor system (with forward path gain)

It can be seen in Fig. 4 that the open-loop system is unstable which can be attributed to the fact that one of the poles of the system is located at the right hand plane (RHP) of the s-plane. It seems that the magnitude of the conveyor speed deteriorates as the operating time increases. In Fig. 5, a full state feedback gain is added to the loop to improve the performance of the system. It can be seen that the system does not meet the step well at all. Not only is the magnitude of the speed response to tracking the desired speed, but also negative instead of positive with a peak amplitude of -1.6×10^{-5} at time 0.05s. In order to address this challenge, the addition of a forward path gain becomes necessary. With this done, a simulation conducted shows as in the plots in Fig. 6, that the step response has been tracked reasonably well. The time domain performance parameters obtained by the proposed system are: rise time of 0.05 s, peak time of 0.17 s, peak overshoot of 4%, and settling time of 0.95 s.

The time domain performance parameters obtained with respect to step forcing input applied to the system indicated that the developed system was able to meet the design criteria chosen in this paper.

CONCLUSION

The paper has presented the design of a full state feedback controller for a DC motor based industrial conveyor system. The results from the conveyor system modeled in state space form and simulated in terms of step response revealed that the controller was able to improve the speed response performance by providing a settling time of 0.95 s and overshoot of 4% to unit step forcing input. Generally, it is obvious that simulation results indicated that with the designed full state feedback controller, the performance of the motor driven industrial conveyor is greatly enhanced in terms of rise time, peak time, percentage overshoot, and settling time.

REFERENCES

- [1] Ufot, O.N., Ekpoudom, I.I., & Akpan, E.A. (2017). Development of a Fuzzy Logic Controller for Industrial Conveyor. *American Journal of Science, Engineering and Technology*, 2(3), 77-82.
- [2] Umorn, M.A., Essien, A.O., & Ekpoudom, I.I. (2016). Design and Implementation of Conveyor Line Speed Synchroniser for Industrial Control Applications: A Case Study of Champion's Breweries PLC UYO. *Nigerian Journal of Technology*, 35(3), 618-626.
- [3] Mohdshah, M., Rahmat, M., Danapalasingnam, K.A., and Abdulwahab M. (2013). PLC Based Adaptive Fuzzy PID Speed Control of DC Belt Conveyor System, *International Journal on Smart Sensing And Intelligent Systems* 6(3) 1271-1278.
- [4] Wahyudie, A., and Kawabe, T. (2010). Characterization of all robust pid controllers for belt conveyor system via corrected polynomial stabilization. *Research Reports on Information Science and Electrical Engineering of Kyushu University* 15, (1), 1-6.
- [5] Selezneva, A. (2007). Modeling and Synthesis of Tracking Control for the Belt Drive System. A

Master of Science Thesis, Department of Electrical Engineering Lappeenranta University of Technology, 2(6), 29-31.

- [6] Rowell, D. State-Space Representation of LTI Systems. 2.14 Analysis and Design of Feedback Control Systems, 1-18.
- [7] Nagrath, I. J., & Gopal, M. (2005). Control Systems Engineering. 4th Edition, New Age International Publishers, 570–640.
- [8] Ogata k.(2002). Modern Control Engineering, 4th ed., Upper Saddle River, NJ: Prentice Hall, pp. 964