Waveform Design for MIMO Radar Systems

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Abstract- Waveform design is critical to the realization of a Multiple Input Multiple Output (MIMO) radar system. If the waveforms being transmitted are perfectly orthogonal, the virtual array consists of more elements than the transmit array and this provides additional degrees of freedom which improves performance. The correlation properties of the wavforms transmitted determine the characteristics of the system. In this paper, a binary orthogonal waveform with low autocorrelation and cross correlation properties is designed. Orthogonality in transmitted waveforms is required in MIMO radar systems to enable the use of match filtering at the outputs to separate the different transmit paths. Exploiting the orthogonality of the walsh hadamard matrix based on non-identical walsh functions, the simulated annealing statistical optimization tool is used to obtain the orthogonal signal set with the desired low correlation properties.

Indexed Terms- Autocorrelation, Multiple Input Multiple Output (MIMO) radar, Match filtering, simulated annealing, Virtual array.

I. INTRODUCTION

Waveform diversity is one of the benefits derived from MIMO radar systems. Multiple diverse waveforms can be transmitted simultaneously for the purpose of improving detection, target classification and parameter identifiability. Waveform design methods can be categorized into the covariance matrix approach, ambiguity function approach, mutual information, and the method of directly designing the time series transmitted from each transmitter. In the covariance matrix method, the covariance matrix is designed to either focus the beam to transmit power to a desired range of angles or to control the spatial power [1]. Mutual information approach relies on prior knowledge of the target to choose waveforms which optimize the mutual information between the received signals and the target impulse response [2]. Another method is the application of numerical optimization techniques to obtain orthogonal waveforms. For rather systems that use match filtering to extract the target signal, the response at the match filter output determines the resolution [3]. This response is characterized by the ambiguity function when doppler effects are also considered. The focus of this paper is on the design details of the time series for orthogonal binary signals transmitted from each transmitter. For small Doppler shifts, the Doppler effect on the time series is negligible, and the response at the output of the match filter becomes the autocorrelation function. Therefore the time series can be designed for a good autocorrelation and cross correlation characteristics. In radar systems, pulse compression is used to achieve the benefits of a short pulse by squeezing a long duration pulse into a short pulse and at the same time retaining the energy of the long pulse. A short pulse requires a large bandwidth and hence can be interference to other users of the band. The shorter the pulse the more information it contains and hence more demands on processing. If the transmitter peak power is low, the shorter the pulse the less energy is transmitted. This makes short pulse radars range limited. With pulse compression, waveforms can be designed to have both long duration and small duration. A small duration waveform is produced when a long duration binary phase coded waveform is applied to a match filter, and hence obtain good detection performance and accurate range measurements [4]. Two important modulations used for pulse compression are linear frequency modulation (LFM) and phase code modulation. Waveforms based on other modulation methods include frequency hopping and polyphase coding. The compressed pulse consists of a desired response and undesired side responses called sidelobes. These sidelobes must be suppressed, so that they are not mistaken as weak targets. Binary sequences with low autocorrelation sidelobe levels and low crosscorrelation peaks have been investigated in [5] In this work, Deng used the statistical simulated annealing algorithm to obtain an optimal set of binary sequences that satisfies the desired correlation properties for the radar system.

In MIMO radar systems, that transmit orthogonal waveforms simultaneously from several antennas and uses match filtering to extract the target echoes, a narrow impulse like autocorrelation function with low sidelobes which reduces interference from other targets is desired. This also ensures a high target resolution, high range resolution and a high SNR. A low crosscorrelation between the transmitted signals enables independent target information from different angles, thus improving the detection of targets with weaker echoes.[6] High cross correlation sidelobes (High spatial sidelobes) causes interference between signals from different directions, thereby compromising angle estimation accuracy. The use of match filtering at the receiver outputs further ensures low crosscorrelation as well as high SNRs [7] The MIMO radar waveform design problem becomes that of designing orthogonal matrices of arbitrary number of rows and columns with good correlation properties. The walsh functions are used to provide the required orthogonality between the signals and the simulated annealing algorithm is used for its effectiveness in discrete and combinatorial optimization problems, to obtain the optimal set of orthogonal signals with the desired autocorrelation and cross correlation properties.

II. FORMULATION OF THE SIGNAL CORRELATION PROBLEM

Consider a MIMO radar system consisting of M-elements at the transmit array. The transmitted waveforms are orthogonal to each other. At the receiver, let there be M_r receive antennas and let the sampling rate at the receiver be equal to that of the transmitter. Assume that each waveform employ N subpulses represented by a complex number sequence, The m^{th} transmitted constant-modulus signal is of the form:

$$\left\{s_m(n) = e^{j\phi_m(n)}, n = 1,2,\ldots N\right\}, m = 1,2,\ldots M \quad (1)$$

Where ϕ_m is the phase of subpulse n of signal m in the signal set, For Binary sequences, the phase for a subpulse alternates between 0 and π represented as -1 and +1 respectively in the sequence (Sun et al., 2010) The signal set can be represented as follows:

$$\mathbf{S}(M,N) = \begin{bmatrix} s_1(1), & s_1(2), & \cdots, & s_1(N) \\ s_2(1), & s_2(2), & \cdots, & s_2(N) \\ \vdots & \vdots & \cdots, & \vdots \\ s_M(1), & s_M(2) & \cdots, & s_M(N) \end{bmatrix} \quad (2)$$

The rows of $S \in \mathbb{C}^{M\times N}$ are the binary modulating code sequences which can be either +1 or -1, and are the transmitted signals or waveforms. The subpulse train will have a much smaller peak power than a single subpulse, at the same total transmitted energy. Also, using constant modulus signals ensures the radar system can use non linear amplifiers, as most MIMO radar systems tend to operate in the saturation region in order to increase range resolution performance. The received data matrix for a particular range bin of interest is denoted as [8].

$$\mathbf{Y} = \sum_{p=1}^{P_0} \beta_{p,0}\mathbf{a}_r\mathbf{a}_t^T\mathbf{S} + \mathbf{E} + \mathbf{W} \quad (3)$$

And $$\mathbf{E} = \sum_{k=-K,k\neq 0}^{K} \sum_{p=1}^{P_k} \beta_{p,k}\mathbf{a}_r\mathbf{a}_t^T\mathbf{S}\mathbf{J}_k \quad (4)$$

Where P_0 represents the number of scatterers in the range bin of interest at lag k = 0. For simplicity, we consider a jammer free model, thus W is an independent and identically distributed (i.i.d.) noise matrix, and $\mathbf{W^H} \approx \sigma^2 \mathrm{I}_{M_R}$

E is the interference term which represents impinging signals from P_k scatterers within $\{-K\ldots\ldots,-1,1\ldots\ldots,K\}$ from neighboring range bins surrounding the range bin of interest. E should be weakened by the filtering process at the receiver, and consequently suppress the range sidelobes. $\beta_{p,k}$ are the complex amplitudes. $\mathbf{a}_r$ and $\mathbf{a}_t$ are the receive and transmit steering vectors respectively. $\mathbf{J}_k$ is a N x N range shifting matrix that takes into account the different propagation times of the signals reflected by neighboring scatterers. The $(i, i+k)^{\text{th}}$ element is one and the others are zeros.

$$\mathbf{J}_k = \begin{bmatrix} \mathbf{0}_{(N-k)\times k} & \mathbf{I}_{N-k} \\ \mathbf{0}_{k\times k} & \mathbf{0}_{k\times(N-k)} \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 1 \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix} = \mathbf{J}_{-k}^T \quad (5)$$

k, which is the lag in the auto correlation and cross correlation function is considered here as the difference in additional sampling intervals between the current range bin and the interfering range bin. Pulse compression leads to

$$\begin{aligned} \mathbf{Z} &= \mathbf{Y}\mathbf{S}^H \\ &= \sum_{p=1}^{P_0} \beta_{p,0} \mathbf{a}_r \mathbf{a}_t^T \mathbf{S}\mathbf{S}^H + \sum_{k=-K,k\neq 0}^{K} \sum_{p=1}^{P_k} \beta_{p,k} \mathbf{a}_r \mathbf{a}_t^T \mathbf{S}\mathbf{J}_k \mathbf{S}^H + \mathbf{W}\mathbf{S}^H \end{aligned} \quad (6)$$

Let's define the *M* x *M* waveform correlation matrix as

$$\mathbf{R}_k = \begin{bmatrix} R_{1,1}(k) & R_{1,2}(k) & \cdots & R_{1,M}(k) \\ R_{2,1}(k) & R_{2,2}(k) & \cdots & R_{2,M}(k) \\ \vdots & \vdots & \ddots & \vdots \\ R_{M,1}(k) & R_{M,2}(k) & \cdots & R_{M,M}(k) \end{bmatrix} \quad (7)$$

Where

$$R_{m,q}(k) = \begin{cases} \sum_{n=1}^{K-k} s_{m,n} \left(s_{q,n} + k\right)^* & 0 \le k \le K-1 \\ \sum_{n=1}^{K+k} s_{m,n-k} \left(s_{q,n}\right)^* & 1-K \le k < 0 \end{cases} \quad (8)$$

is the waveform correlation function between s_m and s_q. In the case where **E** is due to the presence of clutter, the matched filter (MF) maximizes the signal-to-noise ratio (SNR), but it does not maximize the signal-to-clutter ratio (SCR) [9]. The SCR for MIMO radar is defined as

$$\text{SCR} = \sum_{\substack{k\neq 0 \\ k=1-K}}^{K-1} \frac{\left\|\mathbf{S}\mathbf{S}^H\right\|^2}{\left\|\mathbf{S}\mathbf{J}_k\mathbf{S}\right\|^2} = \sum_{\substack{k\neq 0 \\ k=1-K}}^{K-1} \frac{\left\|\mathbf{R}_0\right\|^2}{\left\|\mathbf{R}_k\right\|^2} \quad (9)$$

From **Error! Reference source not found.** and **Error! Reference source not found.**, it can be seen that the compression results and SCRs rely on the waveform cross-correlation functions. Match filtering relies on the assumption that the second and third terms in **Error! Reference source not found.** are uncorrelated with **S** and can effectively attenuate the second (out-of- range) term in **Error! Reference source not found.** under this condition. If sidelobes are high a weak scatterer in the range bin of interest will be overshadowed by reflection from a strong scatterer in another bin. It is impossible to achieve strict orthogonality among the transmitted waveforms for all time delays. Therefore waveforms with low autocorrelation sidelobe peaks and low cross correlation peaks are desired. A joint waveform design problem is formulated in [5] as

$$E_2 = \sum_{m=1}^{M} \max_{k\neq 0} \left|R_{m,m}(k)\right| + \lambda \sum_{m=1}^{M-1} \sum_{q=m+1}^{M} \max_{k} \left|R_{m,q}(k)\right| \quad (10)$$

Where $\lambda \ge 0$ is a weighting coefficient between autocorrelation function and cross correlation function. We wish to minimize this cost function subject to

$$R_{m,m}(k) = \begin{Bmatrix} \frac{1}{N} \sum_{n=1}^{N-k} s_m(n) s_m^*(n+k) = 0 & 0 < k < N \\ \frac{1}{N} \sum_{n=-k+1}^{N} s_m(n) s_m^*(n+k) = 0 & -N < k < 0 \end{Bmatrix} \quad (11)$$
$$m = 1, 2, \ldots M$$

And

$$R_{m,q}(k) = \begin{Bmatrix} \frac{1}{N} \sum_{n=1}^{N-k} s_m(n) s_q^*(n+k) = 0 & 0 \le k < N \\ \frac{1}{N} \sum_{n=-k+1}^{N} s_m(n) s_q^*(n+k) = 0 & -N < k < 0 \end{Bmatrix} \quad (12)$$
$$m \neq q, \quad m, q = 1, 2, \ldots M$$

III. BINARY ORTHOGONAL SET FROM WALSH-HADAMARD MATRIX

Walsh functions can be used to create digital waveforms for even and odd values analogous to sines and cosines used in fourier series. They can be considered as a digital fourier series. The walsh functions consists of trains of square pulses (with allowed states being +1and -1). 2^n walsh functions of order n are represented by the rows of the Hadamard matrix when arranged in sequency order. In sequency order each row has one or more (+1 -1) transitions than the preceding row. The first Hadamard matrix is defined as $\mathbf{H}_1 = 1$ and subsequent Hadamard matrices, denoted by $\mathbf{H}_{2p}$ are determined from

$$\mathbf{H}_{2p} = \begin{bmatrix} \mathbf{H}_p & \mathbf{H}_p \\ \mathbf{H}_p & \mathbf{H}_p^c \end{bmatrix}, \quad p > 1 \tag{13}$$

Where $\mathbf{H}_p^c$ is the complement of $\mathbf{H}_p$. The value of $\mathbf{H}_8$ can be seen to be

$$\mathbf{H}_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} \tag{14}$$

The rows of $\mathbf{H}_8$ are defined by the walsh codes $\mathbf{W}(0)$ through $\mathbf{W}(7)$. As can be seen from the matrix, $\mathbf{W}(0)$ is simply a direct current (d.c) level and is usually ignored in most practical applications. Walsh functions with non identical sequences are orthogonal. $\mathbf{H}_{2p}^T \mathbf{H}_{2p} = \mathbf{I}_{2p}$. All the rows of the walsh matrix cannot meet the required design criterion of low autocorrelation sidelobe peaks and low cross correlation peaks. The solution is to choose an optimal set based on the optimization criterion**Error! Reference source not found.**. The waveform design problem becomes that of designing the orthogonal matrix subject to constraints **Error! Reference source not found.** and **Error! Reference source not found.**. For given values of M and N the minimization of **Error! Reference source not found.** results in an M x N matrix that is automatically constrained by **Error! Reference source not found.** and **Error! Reference source not found.**. However, the Walsh matrix is a square matrix of dimensions (N x N) where N is a power of 2. Selecting M rows from N rows means the matrix is no more square and $\mathbf{S^T S} = \mathbf{I}$ and $\mathbf{SS^T} = \mathbf{I}$ are not equivalent. But since $N > M$, the rows are still orthonormal, $\mathbf{SS^T} = \mathbf{I}$, which is the requirement for orthogonality in MIMO radar.

IV. DESIGN OF SIGNAL SET USING SIMULATED ANNEALING ALGORITHM

Annealing in thermodynamics is the process of heating metals to a high temperature followed by controlled cooling so that the particles rearrange themselves towards lower energies to form a crystalline structure [10] extended the original metropolis scheme to large scale combinatorial optimization problems. The current state of the thermodynamic system is analogous to the current solution to the combinatorial problem, while the energy equation is analogous to the objective function. The ground state, (Temperature, T = 0) is analogous to the global minimum, which corresponds to the optimized sequence set or signal matrix with the lowest autocorrelation sidelobe peaks and cross correlation peaks. The algorithm begins by setting T to a high value and then decremented at each step following an annealing schedule that ends at T = 0. At each step the current configuration is perturbed and the change in energy dE is computed. If the change in energy is negative, the new configuration is accepted. If the change in energy is positive it is accepted with a probability given by the Boltzmann constant, $\exp(-{}^{dE}\!/_{T})$. This prevents the algorithm from being stuck at a local minimum that is worse than the global minimum as the case with the traditional greedy algorithms. The optimization problem is to estimate the best M rows from the (N x N) Walsh-Hadamard matrix, $\mathbf{H}_{2p}$ to form the matrix **S.**

The algorithm steps can be summarized as follows:

- Set the initial temperature, T to a high value or infinity
- Perturb the N x N matrix $\mathbf{H}_{2p}$ by randomly swapping columns and randomly selecting M rows to form the matrix $\mathbf{S}_{\text{current}}$ of dimensions M x N.
- Compute the corresponding cost function $\mathbf{E}_{\text{current}}$ from (10)
- Randomly swap columns and select M rows to form the next matrix $\mathbf{S}_{\text{new}}$
- Compute the corresponding cost function value $\mathbf{E}_{\text{new}}$

- If $\mathbf{E}_{new} \le \mathbf{E}_{current}$, then $\mathbf{S}_{current} = \mathbf{S}_{new}$
- If $\mathbf{E}_{new} > \mathbf{E}_{Current}$, then $\mathbf{S}_{current} = \mathbf{S}_{new}$ only if $\exp\left(\frac{E_{Current} - E_{New}}{T}\right) > rand$
- Annealing schedule; Reduce the temperature $T_{i+1} = \alpha T_i \quad (0 < \alpha < 1)$ where α is constant chosen in this design to be 1
- Repeat steps 3 to step 8 i times until cost function reaches a global minimum and T=0

V. SIMULATION RESULTS

In this simulation, we vary the number of transmit signals M and the sequence length N and obtained the cross correlation and Auto correlation of the optimized set as shown in Figs.1 and 2. Secondly, for an optimized set of dimensions M x N, we chose M=4 which represents the number of transmitted signals and N = 256 the signal length, a maximum autocorrelation sidelobe peak of 0.1397 or -17.09 dB and maximum crosscorrelation peak of 0.1521.or -16.36dB were obtained. These values are normalized with respect to the signal length N. and the design results are single realizations obtained using a dual core Intel processor. From Fig.1 and Fig.2, it is observed that lower correlation values are obtained as the sequence length increases. A lower maximum autocorrelation sidelobe peak means increased detection capability and a reduced maximum cross correlation peak means interference is reduced.

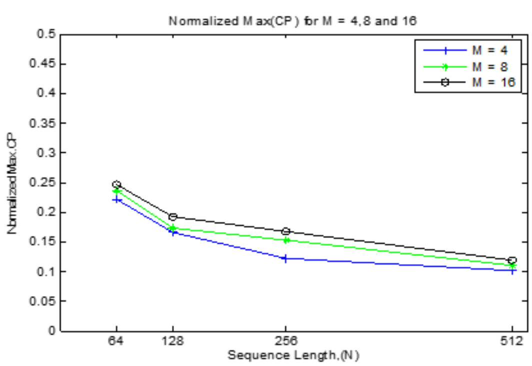

Fig.1 Normalized Cross correlation Peaks

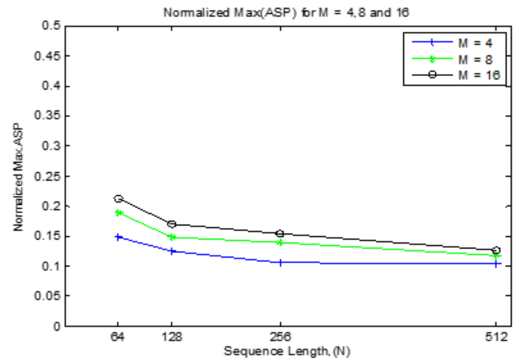

Fig. 2 Normalized Auto correlation Sidelobe Peaks

The results show that the algorithm is more effective for larger code lengths. Fig 3 shows the autocorrelation functions of each transmitted signal in the main diagonal of the figure and the crosscorrelation between signals in the off diagonal of the figure. This illustration of the covariance matrix of the waveforms indicates that if used in a MIMO radar system, target echoes can be clearly visible and interference can be quite negligible. The value $\lambda = 1$ is chosen in the cost function **Error! Reference source not found.** for all optimizations. Fig.4 shows the Magnitude squared plot for the designed binary codes while Fig.5 shows the Magnitude squared plot for a polyphase sequence from the designed polyphase code set in [11] for length, N =40 , size, M = 4 and distinct number of phases, L = 4. The synthesized results show almost equal performance with known polyphase sequences [12] as shown in the Magnitude squared plots of Fig. 4 and Fig.5. as well as Table I.

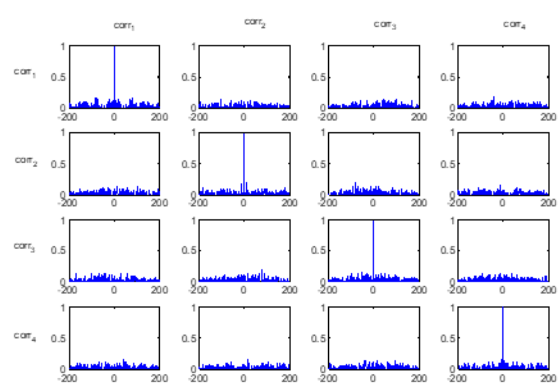

Fig.3 Illustration of the covariance Matrix of a MIMO radar waveform set for M = 4 Orthogonal codes of length N = 256

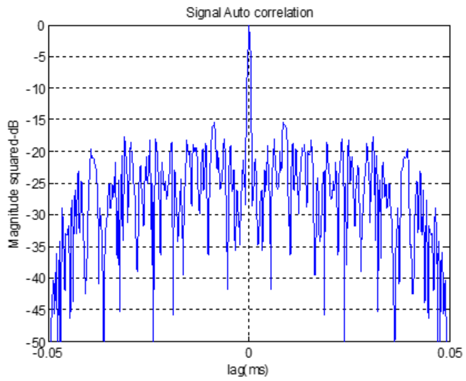

Fig.4 Magnitude squared plot of the Auto correlation of designed Binary codes

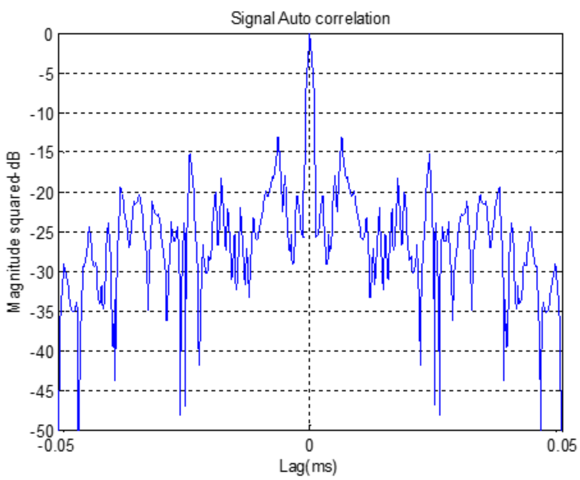

Fig.5 Magnitude squared plots for a sequence length N = 40, and distinct number of phases, L = 4 from a polyphase sequence set in [11].

Table I Comparison between Polyphase Sequence Values [12] and obtained results

	Max(ASP)	Max(CP)
Polyphase values for M = 4, N = 128	0.0954	0.1182
Obtained Binary values for M = 4, N = 128	0.1255	0.1656

VI. CONCLUSIONS

In this paper, we presented a waveform optimization method for multiple-input multiple-output (MIMO) radar systems for good autocorrelation and crosscorrelation properties based on the direct design of the set of orthogonal transmitted time series. Exploiting the orthogonality of non-identical walsh functions, which have been used successfully in spread spectrum communication, the Walsh-Hadamard matrix is used as the orthogonal binary signal set followed by the simulated annealing statistical optimization tool to provide an optimal orthogonal binary signal set with the desired correlation properties. We have investigated the effectiveness of the simulated annealing algorithm for waveform optimization of an orthogonal set of binary waveforms for MIMO radar from the Walsh-Hadamard matrix. As the code length increases both the average auto correlation peaks and average cross correlation peak decreases. This equals the results of known polyphase sequence designs.

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