Rolle's Theorem

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Abstract -- The purpose of this paper is to report on an experiment carried out to study twelfth grade students' understanding of Rolle's Theorem and its relationship to the closely related Mean Value Theorem. In particular, I set up tasks designed to study the learner's ability to state the theorem and apply it to reasoning tasks, the influence of concept images in his or her reasoning about the theorem, and the learner's ability to perceive the relationship between Rolle's Theorem and other related mathematical concepts.

I. INTRODUCTION

Advanced mathematical ideas are characterized by complicated interactions between intuitive and rigorous reasoning processes (Weber & Alcock, 2004). [2]Learning calculus, that involves processes bearing on advanced mathematical thinking, has been a theme of intensive analysis. one among the many conclusions arising out of this analysis is that students generally develop routine techniques Associate in Nursingd artful skills instead of an understanding of theoretical ideas (Berry & Nymann, 2003; Davis & Vinner, 1986; Ervynck, 1981; Parameswaran, 2007; Henry Martyn Robert, 1982; Sierpinska, 1987). The topic of calculus is wealthy in abstraction and demands a high level of abstract understanding, wherever several students have difficulties. Ferrini-Mundy and Graham (1991) argue that students' understanding of central ideas of calculus is 'exceptionally primitive. whereas teaching calculus to a good vary of scholars, it's additional sensible to charm to students' intuition once transfer mathematical ideas and ideas, building on what they need already learned while not creating serious demands on their power for abstract and rigorous mathematical understanding. Some researchers argue that Associate in Nursing introductory calculus course ought to be informal, intuitive[1], and abstract, primarily based principally on graphs and functions (Koirala, 1997); formulas and rules ought to be fastidiously

And intuitively developed on the premise of students' previous add arithmetic and different sciences (Heid, 1988; Orton 1983). one among the guiding principles of teaching calculus can be the 'Rule of 3,' (Hughes-Hallet, et al., 1994) that says that, whenever attainable, topics ought to be tutored diagrammatically, numerically, and analytically. The aim is to balance all 3 of those elements to change the scholars to look at concepts from totally different standpoints and develop a holistic perspective of every idea. There has been in depth analysis into the difficulties that students encounter in understanding limits, functions, differentiability, continuity, and so on. However, there's not abundant literature on students' understanding of different ideas in calculus. with the exception of the psychological feature obstacles that arise within the learning of calculus ideas because of the complexness of the topic matter, students generally encounter difficulties[1] inherent in mathematical reasoning. as an example, abstract thought may be a elementary tool for mathematical thinking; but, students reveal serious difficulties developing such reasoning skills. Orsega and Sorizio (2000) propose the mental model theory of Johnson-Laird and Byrne as a psychological feature framework to investigate students' difficulties in abstract thought. Orsega and Sorizio argue that a didactic model ought to be designed to change undergraduates to beat the fallacies of their deductive inferences. They take into account a pedagogy that permits freshman undergraduates to form express the tautologically implicit properties within the hypothesis of Rolle's Theorem and to mirror on them.

II. WHY ROLLE'S THEOREM?

As ascertained by Berlin ski (1995), [3] "Rolle's Theorem is regarding functions, so a theorem regarding processes depicted by functions, associate affirmation among different things regarding the coordination of your time and house. The constraints alter the 2 basic mathematical properties of continuity and differentiability". Berlinski any observes that: "Rolle's Theorem establishes a association between continuity and differentiability. Continuity guarantees a maximum; differentiability delivers variety. Fermat's Theorem [which says that if f contains a native political orientation at c and if f'(c) exists, then f'(c) = 0] provides the association between concepts". The statement of the theory involves multiple (for all) and therefore the hypotheses, the quantifier (there exists). Additionally Rolle's quantifier Theorem offers the chance for pictorial, intuitive, and logical interpretations. The data parts needed for the understanding of this theorem involve limits, continuity, and differentiability. The proof of the theory is given mistreatment the Fermat's Theorem and therefore the Extreme price Theorem, that says that any real valued continuous perform on a interval attains its most and minimum values. The proof of Fermat's Theorem is given within the course whereas that of maximum price Theorem is taken as shared (Stewart, 1987). thus we have a tendency to attractiveness to the learners' intuition instead of be rigorous in our approach. the sweetness of this theorem additionally reveals itself in its reference to world[5]. A ball, once thrown up, comes down and through the course of its movement; it changes its direction at some purpose to return down. Rolle's Theorem so will be accustomed make a case for that the rate of the ball that is thrown upwards should become zero at some purpose (Berlin ski, 1995). Theoretical Background we have a tendency to shall use the ideas introduced by Weber and Alcock (2004) in their analysis on syntactical and linguistics proof production. a symptom is named as syntactical if it involves solely manipulation of facts and formal definitions in an exceedingly logical manner while not appealing to intuitive and non-formal representations of the mathematical ideas concerned. The prove would like solely to own the flexibility to create formal deductions supported the relevant definitions and ideas. Such data and understanding is termed syntactical data or formal understanding. On the opposite hand, if the prove uses instantiations to guide the formal inferences, he or she is claimed to possess linguistics or effective intuitive understanding. internal representation is delineated as "a consistently repeatable method that a personal cares a mathematical object, that is internally meaty thereto individual". "Semantic or effective intuitive understanding is delineated because the ability on the a part of the evidenced expressly describes however she may translate intuitive observations supported instantiations into formal mathematical arguments". whereas formal understanding is at the superficial level, effective intuitive understanding lies at a deeper level and is characterized by the subsequent options, that we have a tendency to illustrate within the context of Rolle's Theorem: • internal representation "One ought to be ready to instantiate relevant mathematical objects". for instance, within the case of understanding Rolle's Theorem, the learner instantiates the statement of the theory if he acknowledges its pertinence (or non-applicability) within the case of a "typical" perform -- not simply that of a quadratic-- presumably in terms of a graph.

Richness:-"These instantiations ought to be wealthy enough that they counsel inferences that one will draw".

Accuracy: - "These instantiations ought to be correct reflections of the objects and ideas that they represent".

In our context, the examples shouldn't be too special on counsel properties not implicit by the theory. for instance, one could also be misled to believe that there's a novel purpose wherever the spinoff vanishes if one continually instantiates the graph to be a conic section.

The tangent at t is parallel to the secant connecting the points (g, f (g)) and (h, f (h)) . This suggests the alternative notation

$\Delta y = f'(t)\Delta x$ for describing the mean value theorem.[4]

The function

$$g(x) = f(x) - f(g) - \frac{f(h) - f(g)}{h - g}(x - g)$$

Vanishes at x = a and x = b Since f is continuously differentiable, by Rolle's theorem, there exists to(a, b) with[6]

$$0 = g'(t)$$

$$= f'(t) - \frac{f(h) - f(g)}{h - g}$$

i.e.

$f(h) - f(g) = f'(t)(h - g)$

III. GENERALIZED ROLLE'S THEOREM

Let f be continuous on [g, h] and n times differentiable on (g, h). If f is zero at the 1+n distinct points $x_0 < x_1 < \ldots < x_n$ in [g, h] then there exists a number c in (g, h) such that $f^{(n)}(c) = 0$

Consider a real-valued[1], continuous function *f* on a closed interval [g, h] with *f*(*g*) = *f*(h). If for every *x* in the open interval (g, h) the right-hand limit

$$f'(x^+) := \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h}$$

and the left-hand limit

$$f'(x^-) := \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h}$$

exist in the extended real line [−∞, ∞], then there is some number *c* in the open interval (g, h) such that one of the two limits

$$f'(z^+) \quad \text{and} \quad f'(z^-)$$

is ≥ 0 and the other one is ≤ 0 (in the extended real line). If the right- and left-hand limits agree for every *x*, then they agree in particular for *c*, hence the derivative of *f* exists at *c* and is equal to zero.

Proof:

Since the proof for the standard version of Rolle's theorem and the generalization are very similar, we prove the generalization.

The idea of the proof is to argue that if *f*(*a*) = *f*(*b*), then *f* must attain either a maximum or a minimum somewhere between *a* and *b*, say at *c*, and the function must change from increasing to decreasing (or the other way around) at *c*. In particular, if the derivative exists, it must be zero at *c*. By assumption, *f* is continuous on [*a*, *b*], and by the extreme value theorem attains both its maximum and its minimum in [*a*, *b*]. If these are both attained at the endpoints of [*a*, *b*], then f is constant on [*a*, *b*] and so the derivative of *f* is zero at every point in (*a*, *b*).

Suppose then that the maximum is obtained at an interior point *c* of (*a*, *b*) (the argument for the minimum is very similar, just consider −*f*). We shall examine the above right- and left-hand limits separately.

For a real *h* such that *c* + *h* is in [*a*, *b*], the value *f*(*c* + *h*) is smaller or equal to *f*(*c*) because *f* attains its maximum at *c*. Therefore, for every *h* > 0,

$$\frac{f(c+h) - f(c)}{h} \leq 0,$$

Hence

$$f'(c^+) := \lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h} \leq 0,$$

Where the limit exists by assumption, it may be minus infinity.

Similarly, for every *h* < 0, the inequality turns around because the denominator is now negative and we get[3]

$$\frac{f(c+h) - f(c)}{h} \geq 0,$$

Hence

$$f'(c^-) := \lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h} \geq 0,$$

where the limit might be plus infinity. Finally, when the above right- and left-hand limits agree (in particular when *f* is differentiable), then the derivative of *f* at *c* must be zero. (Alternatively, we can apply Fermat's stationary point theorem directly.)

IV. EXAMPLES

Example 1:-

The graph of $f(x) = - x^2 + 6x - 6$ for $1 \leq x \leq 5$ is shown below. f(1) = f(5) = - 1 and f is continuous on [1, 5] and differentiable on (1 , 5) hence, According to Rolle's theorem, there exists at least one value of x = z such that f '(z) = 0.
f '(x) = - 2 x + 6
f '(c) = - 2 z + 6 = 0
Solve the above equation to obtain[3]
z = 3
Therefore at x = 3 there is a tangent to the graph of f that has a slope equal to zero as shown in figure 1 below.

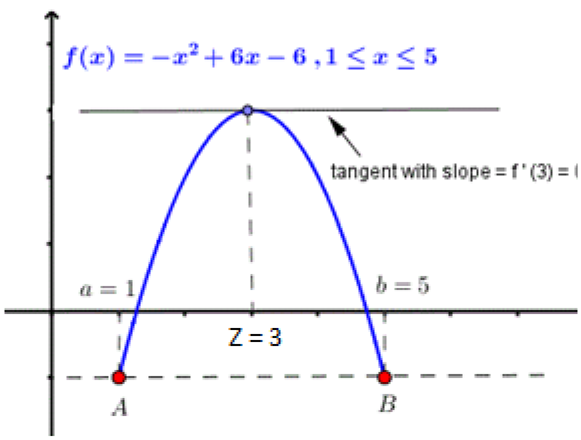

Fig 1.1 Graph with slope equal to zero

Solution:-

f(0) = 1 and f(2π) = 1 therefore f(0) = f(2π) f is continuous on [0 , 2π] Function f is differentiable in (0 , 2π) Function f satisfies Rolle's theorem
function g has a V-shaped graph with vertex at x = 2 and is therefore not differentiable at x = 2
Function g does not satisfy Rolle's theorem
Function h is undefined at x = 0. Function h does not satisfy Rolle's Theorem.

dk(x) = |sin(x)| , for x in [0 , 2π] The graph of function k is shown below and it shows that function k is not differentiable at x = π. Function k does not satisfy Rolle's theorem[2].

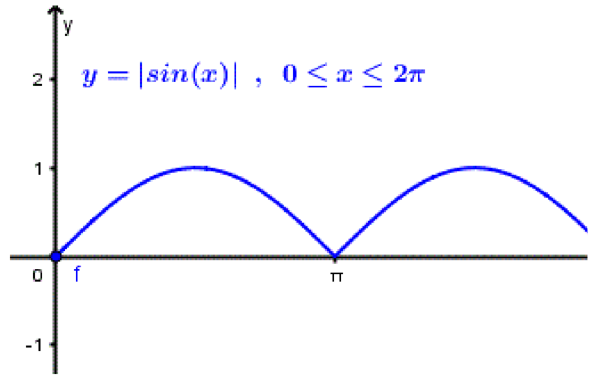

Fig. 1.2 Graph of Function k (differential at x= a)

V. CONCLUSION

We aimed to review students' understanding of Rolle's Theorem by putting in specific tasks that Revathy Parameswaran twenty five concerned stating the concept, relating it to the norm Theorem, and victimization it to resolve issues involving graphs. The instantiations within the context of Rolle's Theorem appear to involve, in most cases, acquainted functions or graphs like the conic. For a few students, this ends up in a misunderstanding or interpretation of the hypothesis or conclusion of the concept. Their instantiations area unit too specialized they lack richness and accuracy.

To summarize, our experiment reveals potential difficulties students have in understanding Rolle's Theorem, that involves creating sense of it, concerning alternative ideas like the worth|mean|average|norm} Theorem and intermediate value theorem, furthermore because the ability to use it in things once the operate isn't expressly given.

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