© DEC 2019 | IRE Journals | Volume 3 Issue 6 | ISSN: 2456-8880

Obtaining Roots of Non-Linear Equation Using Newton Raphson Method

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Abstract- This review paper is based on the Topic of Newton Raphson Method in MATLAB. It begins with the use of Newton Raphson method in finding the approximate root of the given equations, how it is better over other methods, its applications, and limitations and how limitations are overcome.

Indexed Terms- Newton Raphson Method, Convergence, Efficient, Approximate, Limitations, Application,Overcome.

I. INTRODUCTION

Newton Raphson Method is named after the two scientist (Isaac Newton and Joseph Raphson). It is a root finding algorithm with the better approximation to the roots of the real-valued function. This method is well known for its fast rate of convergence. Since, root finding has been one of the problems in the practical applications, Newton's method is very fast and efficient way as compared with the other methods. This method requires only one iteration and the derivative evaluation per iteration. If compared the rate of convergenceofBisection, Newton and Secant method the result is obtained as Bisection < Newton < Secant [1].Estimating roots of the non- linear equation with the help of Newton Raphson method provides better result with fast convergence speed and MATLAB has also adopted this method for finding the roots.

II. NEWTONRAPHSON METHOD

2.1 Derivation

Geometrically, (x1, 0) is the intersection with the x-axis of the tangent to the graphof f at (x0, f (x0)). The basic version starts with a single variable function *f* defined for a realvariable *x* the function's derivative *f '*and an initial guess *x0* for a root of*f*.

If the function satisfies sufficient assumptions and the initial guess is close [2], then

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Here, x1is the better approximation to the root as compared with the initial guess x0.The method is continued to n number of iterations until sufficiently precise value is obtained. The general formulae for successive iterations is givenas:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Where, (n=0, 1, 2, 3, 4…)

Newton Raphson Method provides a good result in finding the roots of a non-linear equation with fast convergence speed and MATLAB has also adopted this technique.

- Examples

Suppose that we have to find the square root of 612 and let the square of x be 612 i.e,

x^2 =612

Or we can write the function for Newton's Method,

$f(x) = x^2 - 612$

With derivative,

f'(x) = 2x

Now assume 10 as our initial guess (x0=10) and then Newton's iterations series will be given as:

x1= x0 – f(x0) / f'(x0)

x1= 10 – 10^2 - 612 / 2.10 = 35.6

Further, iteration values are represented in the tabular formas:

Iterartion No.	Xn+1	Values
1.	X_1	35.6
2.	X_2	26.39550561
3.	X_3	24.79063549
4.	X_4	24.78965477
5.	X_5	24.75932547
6.	X_6	24.73898751
7.	X_7	24.73863375

Having only few iterations one can find the root with better approximation and upto as many decimal places as required.

III. APPLICATIONS

3.1 Finding maxima or minima

As the derivative of function at maxima orat minima is zero therefore, this method can be

3.2 Finding square roots

This method is also very useful in obtaining the square root of numbers correct upto as many decimal places. Ex- $2^{1/2}$ = 1.41421356237309504880 is obtained upto desired number of decimal places by solving few number of iterations [3].

3.3 Solvingtranscendental equations

Many transcendental equation can be solved with the help of newton's method. Given,

g(x)=h(x)

where, g(x) and/or h(x) are/is transcendental. Then, f(x) = g(x)-h(x)

[4] solving above equation for x with the help of Newton's method transcendental equation can be solved.

IV. LIMITATIONS

Newton's method will be applicable only if the assumptions made in the proof of quadratic convergence are met, However if this assumptions are not metthe method would not converge. Following points imposes limit on the use of this method

4.1 Bad startingpoints

In some cases the necessary conditions are met but the point chosen as an initial point is not in the interval and so the method not converges and it becomes hard to find the solution.

4.2 Stationary iterationpoint

used to find local maxima or minima of a function,f(x).

Consider a function,

$f(x) = 1 – x^2$

the function has the maximum value at x=0 and f(x)=0 gives x= 1,-1 as roots. Now if we take x0=0 as initial point then [5],

x1= 0 – 1/0 = ∞ i.e.,x1is not defined and further iterations can't be solved and thus it becomes stationary point for the method. In this case initial point is stationary but in some cases any point at any other iteration can be found as stationary point[6].

4.3 Derivativeissue

Suppose the function with us is not continuously differentiable in the neighbourhood of the root, then the Newton's method will not converge and hence fail [7].

V. HOW LIMITATIONS AREOMITTED

All the above mentioned limitations are overcomed by the use of bisection and secant methods.

VI. CONCLUSION

With the reference to research paper we conclude that how newton method is better of all the other method and how it can be used to find the root of non-linear equation upto desired decimal places. It's faster rate of convergence made it more useful and saves time. We also went through it's various useful applications and some serious drawbacks which put limitation on it's use. Also we discussed how these limitations can be overcomed. We also concluded thattheNewton Raphosn method can be used very effectively to determine the intrinsic value based on its measured permittivity.

ACKNOWLEDGEMENT

We would like to express our special thanks of gratefulness to Dr. D.S. Bankar, Head, Department of electrical engineering for their guidance and support for completing my research paper. I would also like tothank the faculty members of the department of electrical engineering who helped us with extendedsupport.

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