Modeling for Optimization of Product – Mix in Poultry Feeds Production of ABJ Animals Feeds Company

NWOSU MOSES C.[1], AGUH PATRICK S.[2], EZEANYIM, OKECHUKWU C.[3]
[1, 2, 3] *Nnamdi Azikiwe University, Awka*

Abstract- The work done in this investigation is modeling to optimization product mix in industrial settings of resources allocation and utilization in poultry feed production. Most poultry feeds producers in Nigeria do not lend themselves to flexible production process which is important for them to manage the use of resources for effective optimal production. This research focuses on linear optimization for achieving product- mix optimization in terms of the product identification and the right quantity in feed brand production for better profit and optimum firm performance. The computational experiments in this research contains data and information on the units item costs, unit contribution margin, maximum resources capacity, individual products absorption rate and other constraints that are particular to each of the six products produced in the company employed as case study. In data analysis, linear programming model was employed with the aid of software to analyze the data. After solving the problem using MATLAB -LINPROG Optimization software, it was found out that the average monthly profit using the current production plan was less than that of the optimal production plan although the difference was not significant. Therefore, the researcher recommended using the optimal production plan in order to maximize the company's total profit in terms of peso value. The contributions of the study to the host company were: increase in profits; efficient and effective use of resources; and productivity improvement

Indexed Terms- Optimization, Modeling, Poultry Feeds, Product – Mix, Linear programming.

I. INTRODUCTION

The problem that led to this research is for this company "ABJ Animals Feeds Manufacturing & Associated Technologies," over times has been producing and selling on monthly basis, but some brands of feeds are over produced and others are under produced. Despite limited financial and materials resources the company is producing at minimal profits. The problem is to determine the optimal quantities of each brand of these feeds to produce to maximize the monthly profit of the company. The company has six brands of poultry FEEDS in production at 500 bags per brand type.

The task of making effective decision in a modern-day business environment has become so intricate given the rising complexities in the economic and the different socio-political factors. Though, technology changes constantly with new knowledge, new and unprecedented problems also surface by the day. Therefore, operational efficiency must extend beyond optimizing the routine problems to addressing wider and strategic issues. To achieve this, decision makers must look beyond personal experience, intuitive knowledge to avert the serious and costly consequences often associated with wrong decisions. All these and many more have made the application of operational research techniques of fundamental importance to decision- maker (Sharma, 2008).

An essential decision typical of a manufacturing concern is determination of profit maximization of product mix without compromising quality. Thus, in the present competitive environment, the efforts of manufacturing organizations should be towards optimization of all production variables for profitability and better financial performances. In Nigeria, where many organizational entities are managed by nonprofessional but experienced managers, the use of modern managerial tools is limited. However, non-application of modern tools of management in resource allocation, despite the growing number of modern scientific and tested methods of decision making in a global world can be very costly. Resource scarcity is a major constraint in any firm's decision regarding optimal products mix as well as profit maximization (Albright & Wayne, 2009:Arogundade & Adebiyi, 2011). According to Albright & Wayne (2009), these and many other optimization decisions can be well handled by modern linear programming modeling procedures via appropriate problem formulation and equation fittings.

According to Savsar and Aldaihan (2011), the decision-making in product mix selection is difficult due to large number of possibilities involved. Limitation of resource capacities makes it difficult to meet customer demand and hence reducesthe opportunity to earn profits. Several optimization models, including linear programming, have been applied to the product mix selection problem. Since its development in 1947, linear programming (LP) theory and related solution techniques have been widely extended and used as an optimization tool in a variety of problem areas including the production planning and product mix selection areas. However, most of the work involves either theoretical development or hypothetical applications. There is a definite need to apply these theoretical formulations to real life problems. Thus, this is one of those few applications to real life situation of a manufacturing company in Nigeria. Data were sought from the records of the day-to-day activities of the organization for modeling andanalysis.

The technique needs the formulation of the problem and fitting it into a mathematical model which could be run with commercial packages like MATLAB optimization software to get result that will aid optimum decision. Decision-makers are often faced with the problem of determining optimal allocation of limited resources. The problem of determining the best combination of activity levels which does not use more resources than are actually available and at the same time maximize output, revenue, service level, or to minimize costs. However, this research effort explored the benefits of using the Linear Programming techniques for product-mix optimization of poultry feed production in Nigeria. This research tries to answer three basic questions. The basic research questions that prompted this work are:

i. How can we model Animal feeds product- mix problem with linear programming?
ii. Are managers of feeds production industry in Nigeria aware of the existence and the usefulness linear programming for feed product - mix optimization?
iii. How can we recommend the use of linear programming software packages for accommodating large product mix data to Nigeria managers of resources?

To this end, the present work try to explore the application of basic linear programming procedure for product mix optimization in manufacturing organization in Nigeria with the view to establishing how linear programming techniques and the use of MATLAB - LINPROG software could contributes to optimal product mix of poultry feed production. While we are aware that both the method and the software have been applied in analyzing the optimization problems across several industries locally and beyond, there is no study on record that has applied same to the study of optimal product mix in poultry feeds production particularly in Nigeria.

II. LITERATURE REVIEW

Developed in 1939 by Kantorovich and extended by Dantzig (1947), linear programming is a typical mathematical method for resolving multiple constrained optimization (MCO) problems particularly where both the objective and the constraints are linear. Unlike a single constraint (SCO) problem that can be resolve with basic traditional approach, the MCO problems are better handled by modern approach. Interestingly, the usage of this mathematical technique has been widely accepted and facilitated by the advent of computers that can ease complex calculations.

In typical SCO problems involving how to reach a particular isoquant in the face of a defined isocost and vice versa, output maximization and cost minimization can be traditionally attained at the point of tangency between isocost and isoquant. The real-world scenarios require much more with firms facing several constraints either in terms of input sourcing (material adequacy, relevant skilled labour and cutting-edge technology) in the short run or in the form of output specification (quantity and quality requirements). The scope of MCO problems require the use of linear programming since their solutions cannot be obtained from the application of basic traditional approach.

Accordingly, we can describe linear programming as a unique way of solving optimization problems with linear and multi variable objective function where the constraints are linearly equal or unequal.

Linear programming has helped decision makers to set broad objectives and optimize schedules to meet set goals, linear programming and its various extensions like mathematical programming have demonstrated the capacity to solve advanced optimization problems involving both linear and integer constraints. Simply put, the role linear programming in broad mathematical program is typical of that of derivatives of functions in calculus. According to Dantzig and Thapa (1997), LP viewed from the methodical point seeks to identify the extreme of functions further satisfying set of constraints in a bid to rationalize several technological and managerial decisions.

Hillier and Lieberman (2005) describe linear (mathematical) programming as a planning instruments that produces outcomes which best approximates specified objectives amidst all feasible alternatives. They added that linear programming application is not limited to allocating resources to undertaking salone but extend to cover all other firm's issues whose mathematical model fits the very general format for the linear programming model particularly those problems that are multi-faceted. Studies on product mix has been considered in literature, Byrd and Moore, (1978) applied LP model to corporate policy making; Lee and Plenert (1996) worked on what is the best tool for analyzing maximizing product mix Profitability. Also Chung, Lee and Pearn, (2005) studied product mix optimization for semiconductor manufacturing based on AHP and ANP analysis while Savsar and Aldaihan (2011) presented an application of linear programming to product mix selection problem in an oil company. But, as far the researchers are aware; none of the previous studies on LP application to product mix was applied to paint production and more specifically in Nigeria where manufacturing sectors are facing some peculiar challenges. Thus, this study will fill the gap in literature.

III. MATERIALS AND METHODS

3.1. Data and Sources

The major source of data for this study is the personal interview and records of production and operational plan of animal feed product companies in Nigeria. However, in other to have an idea of what is obtainable in a particular company for the purpose of generalization of idea; one of such feed producing companies was chosen as case study. The choice of this organization is hinged on the readiness of this organization to release relevant information for the purpose of this study.

3.2. Data Analysis

This section of the paper analyses, the data collected from personal interview at the company by consulting various operational reports and publication of the animal feed manufacturing company. The interview provided information on the production planning and product mix process techniques used in the organization. This information in addition with the sales and other operating data was analyzed to provide estimates for Linear Programming model parameter. The product mix optimization was obtained by evaluating the mathematical model programming model using MATLAB- LINPROG computer software, the solution of which included the sensitivity analysis.

3.3. Model Nomenclature

The product of typical Animal feeds and company in Nigeria:

Let Xi represent the number of products to be produced at poultry feeds manufacturing Company. Where i range from 1 to 6 represent the following products

Product Labeling:
X1 represents Broiler starter mash
X2 represents Chicks mash
X3 represents Growers mash
X4 represents Broilers finisher mash
X5 represents Layers mash
X_6 represents Supper layers mash

The company produces six (6) types of poultry feeds (Broiler starter mash, Chicks mash, Growers mash, Broilers finisher mash Layers mash, and Supper layers mash). Seventeen inputs are needed here to make six different poultry feeds products. The seventeen inputs are; maize (kg), Sorghum(Dawa) (kg), Soya beans (kg), Fish meal (kg), Groundnut Cake (kg), Palm kernel cake (kg), Wheat Offal (kg), Limestone (kg), Periwinkle (kg), Bone meal (kg), Salt (kg), Premix (kg), Methionine (kg), Lysine (kg), Bio-media (kg), Labour hours per product (hours) and Demand for product.

The outputs are six types of poultry feeds produced at the company. Hence the optimal implementation of these products by the simplex or interior point algorithm will be, to use six (6) decision variables.

Table 1: Quantity Unit Variable Cost, Resources Absorption Rate and Contribution Margin of Each Product

s/n	Material Resources	X_1	X_2	X_3	X_4	X_5	X_6	Maxi. Available Materials, (Kg)
1	Maize	500	500	200	400	452	500	1276000
2	Sorghum	0	50	200	50	100	85	242500
3	Soya bean	200	180	150	200	200	250	590000
4	Fish meal	20	10	0	20	20	30	50000
5	Groundnut cake	50	30	25	50	50	15	110000
6	Palm kernel cake	100	100	200	150	0	0	275000
7	Offal	50	50	200	50	50	50	225000
8	Lime stone	50	30	30	50	40	48	124000
9	Periwinkle	30	20	20	40	40	40	95000
10	Bone meal	20	20	10	20	20	20	55000
11	Salt	3	3	3.5	3.5	4	4	10500
12	Premix	3.5	3	3	3	4	4	10250
13	Methionine	2	2	1	2	2	2	5500
14	Lysine	0.5	0.5	0.5	1	1	1	2250
15	Bio-media	1	1	1	1	1	1	3000
16	Labor per product	1500	1200	1000	1000	1000	1000	6700hr/month
17	Demand for product	1200	2500	2500	1500	3000	1100	120000bags
18	Plant capacity	6000	6000	11000	8000	13000	4000	1720000
19	Warehouse capacity	W_1	W_2	W_3	W_4	W_5	W_6	2500000
20	Unit contribution margin (#)	200	150	180	175	225	200	

3.4. Model Formulation

In the model formulation, the following steps are followed: with reference to the

Data in table 1;

X_1= units of product 1 (Broiler starter mash) produced per month

X_2= units of product 2 (Chicks mash) produced per month

X_3= units of product 3 (Growers mash) produced per month

X_4= units of product 4 (Broilers finishers mash) produced per month

X_5= units of product 5 (Layers Mash) produced per month

X_6 = units of product 6 (Super layer mash) produced per month

Z = total profit per month (in Naira) from producing these six products

Problem Identification: The problem of this study is to maximize profit.

Maximize, Z: $C_1X_1 + C_2X_2 + C_3X_3 + C_4X_4 + C_5X_5 + C_6X_6$(1)

In formulating the model properly, we have:

Objective Function, f

Maximize Z, (# Profit): $200X_1 + 150X_2 + 180X_3 + 175 X_4 + 225 X_5 + 200X_6$

Subject to:

$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$ (Non negativity constraint)
There is an assumption that no negative output can be produced. In formulating the model properly, we have to rewrite the inequality into equation by introduction of slack variables as follows: the slack variables, *a, b, c, d… s,* are added to the equations with inequality to convert them to linear equations as shown below. In each case, the slack variables *a, b, c, d ... s,* are greater than or equal to zero and represent the amount of material available but not used. In this work a model of linear equations were used and represented without slack or surplus variables, present as:

$500X_1 + 500X_2 + 200X_3 + 200X_4 + 500X_5 + 500X_6 = 1276000$

$200X_3 + 50X_4 + 100X_5 = 2\,42{,}500$

$200X_1 + 180X_2 + 150X_3 + 200X_4 + 200X_5 + 2\,50X_6 = 590000$

$20X_1 + 10X_2 + 25X_4 + 20X_5 + 30X_6 = 50000$

$50X_1 + 20X_2 + 25X_3 + 50X_4 + 50X_5 = 110000$

$100X_1 + 100X_2 + 150X_3 + 150X_4 = 275000$

$50X_1 + 50X_2 + 200X_3 + 75X_4 + 50X_5 + 50X_6 = 225{,}000$

$50X_1 + 30X_2 + 30X_3 + 50X_4 + 40X_5 + 45X_6 = 124000$

$30X_1 + 20X_2 + 20X_3 + 40X_4 + 40X_5 + 40X_6 = 95000$

(2)

$20X_1 + 20X_2 + 20X_3 + 10X_4 + 20X_5 + 20X_6 = 55000$

$3X_1 + 3X_2 + 3.5X_3 + 3.5X_4 + 4X_5 + 4X_6 = 10500$

$3.5X_1 + 3X_2 + 3X_3 + 3X_4 + 4X_5 + 4X_6 = 10250$

$X_1 + X_2 + X_3 + X_4 + 2X_5 + 2X_6 = 5500$

$0.5X_1 + 0.5X_2 + 0.5X_3 + X_4 + X_5 + X_6 = 2250$

$X_1 + X_2 + 0X_3 + X_4 + X_5 + X_6 = 3000$

$1.5X_1 + 1.2X_2 + X_3 + X_4 + X_5 + X_6 = 6700$ hr/month

$12000X_1 + 25000X_2 + 26000X_3 + 16000X_4 + 30000X_5 + 11000X_6 = 120000$

X1+ X2+ X3+ X4+ X5+ X6 =120000

3.5. Data Arrangement for Linear Programming (LINPROG) in MATLAB Software

Objective Functions, f: f_1 = [200; 150; 180; 175; 225; 200]

f_2 = [250; 200 ;280; 225; 225; 200]

[500 500 200 500 500 500; 0 0 200 50 100 0; 200 180 150 200 200 250; 20 10 0 25 20 30; 50 20 25 50 50 0; 100 100 200 150 0 0; 50 50 200 75 50 50; 50 30 30 50 40 45; 30 20 20 40 40 40; 20 20 10 20 20 20; 3 3 3.5 3.5 4 4; 3.5 3 3 3 4 4; 2 2 1 2 2 2; 0.5 0.5 0.5 1 1 1; 1000 1000 1000 1000 1000 1000; 1350 1200 1000 1000 1150 1000; 1200 2500 2600 1600 3000 1100]

Variable b = [1276000; 242500; 590000; 50000; 110000; 275000; 225000; 124000; 95000;
55000; 10500; 10250; 5500; 2250; 3000; 6700; 120000]

IV. RESULTS AND DISCUSSIONS

4.1. Results

Software Generated Results were obtained by employing data in section 3.5 to the simplex algorithm of the MATLAB -LINPROG to run the analyses as shown in (a) and (b) below:

4.1.1. LINPROG with Simplex Algorithm Generated Results:

a) Maximize Z, (# Profit): $200X_1 + 150X_2 + 180X_3 + 175X_4 + 225X_5 + 200X_6$

Subject to:

Optimization running.

Objective function value: # 686477.8145695364(Naira)

Exiting: The constraints are overly stringent; no feasible starting point found.

Index	Values
X1	2250
X3	704
X5	1603

X6 316

The cost function of the process is Z (# Profit) = $200X_1 + 180X_3 + 225X_5 + 200X_6$

b) Maximize Z, (# Profit): $250X_1 + 200X_2 + 280X_3 + 225 X_4 + 225 X_5 + 200X_6$

Subject to:

Optimization running.

Objective function value: #774440.5629139075 (Naira)

Exiting: The constraints are overly stringent; no feasible starting point found.

Index	Values
X1	2250
X3	704
X5	1603
X6	316

4.1.2. LINPROG with Interior Point Algorithm Results

Employing data in section 3.5 to the Interior Point Algorithm of the MATLAB - LINPROG to run the analyses as shown in (c) and (d) below:

c) Maximize Z, (# Profit): $200X_1 + 150X_2 + 180X_3 + 175 X_4 + 225 X_5 + 200X_6$

f_1 = [200; 150; 180; 175; 225; 200]

Subject to:

Optimization running

Objective function value: #703475.9609171301(Naira)

Exiting: The primal is infeasible; the equality constraints are dependent but not consistent.

Objective Function, f_1

f_1 = [200; 150; 180; 175; 225; 200]

Index	Values
X1	2250
X2	36
X3	414
X4	149
X5	415
X6	269 ,

Current iteration 6

The cost function of the process is Z (# Profit) = $250X_1 + 200X_2 + 280X_3 + 225 X_4 + 225 X_5 + 200X_6$.

d) Maximize Z, (# Profit): $250X_1 + 200X_2 + 280X_3 + 225 X_4 + 225 X_5 + 200X_6$

Objective Function f_2,f_2 = [250; 200; 280; 225; 225; 200]

Optimization running.

Objective function value:# 963702.2637498571(Naira)

Exiting: The primal is infeasible; the equality constraints are dependent but not consistent.

Index	Values
X1	2250
X2	108
X3	112
X4	956
X5	415
X6	196 Current iteration 6

4.2 Discussion of Results

Considering the results obtained, the results revealed to us that MATLAB - LINPROG by simplex Algorithm took (0) iteration to solve the product mix optimization problem of the case company. More so, that the maximum profit attainable from the six products as it was presented in the above simplex model at f_2 specified is #774440.5629139075 (Nigeria Naira). Furthermore, considering the product – mix in interior point model analysis, the result tells us that MATLAB - LINPROG by Interior Point Algorithm took (6) iteration to solve the product mix optimization problem of the case company. Secondly, that the maximum profit attainable from the six products as it was presented in the above interior point model specified is #963702.2637498571 (Nigeria Naira).

In addition, the quantities of each product to be produced by simplex model: X1- Broiler starter mash, X2- Chicks mash X3 - Growers mash, X4 - Broilers finisher mash, X5 - Layers mash and X_6 - Supper layers mash are 2250, 0, 704, 0, 1603 and 316 respectively. While the quantities of each product to be produced by simplex model: X1- Broiler starter mash, X2- Chicks mash X3 - Growers mash, X4 - Broilers finisher mash, X5 - Layers mash and X_6 - Supper layers mash are 2250, 108, 112, 956, 415 and 196respectively. Interesting thing in these results is the fact that only four out of the six products are profitable in the simplex model, while others were not profitable for production. Thus product X_1, X_3, X_5 andX_6 are responsible for the maximum objective of #686477.8145695364 per month (Nigeria Naira) while other two products still required considerable improvement (reduction in the cost resources) before they can be considered for production. But in the interior point algorithm, all the products are considered profitable in the production, all things being equal and the profitability in the simplex model is less than that of the interior point model by a difference.

However, from the result of data analysis, it was found out that for the organization to operate optimally in the background of inherent constraints and with simplex algorithm, it must produce 2250 bags of product X_1, 704bags of product X_3, 1603bags of product X_5 and 316 bags of products X_6 and zero bags of other two products (X_2 and X_4) for optimum production. While within interior point algorithm, it must produce 2250 bags of product X_1, 108bags of product X_2, 112bags of product X_3, 956 bags of X_4, 415bags of X_5 and 316 bags of products X_6.

The essence of operation research therefore is to minimize waste, one of the beauties of the computer software used is that it went further to postulate possible reduced cost incurred in the other two unprofitable products if the company wants to produce still to achieve maximum profit desired.

From the above premise, it could be inferred that, though un-utilized capacities abound, but if optimality is allowed, the overall objective of reward maximization of the organization is achievable. It is important to state that solution was optimal at second iteration of the Linear programming problem, introducing the interior point algorithm, the other two unprofitable products if the company wants to produce still achieve maximum profit desired. This includes:

a) #21600 (Nigeria naira) on product X_2;
b) # 215100 (Nigeria naira) on product X_4;

However, it was of greater interest to note also that, organizations should not only crave for maximization, but optimality. The implication of this is that, reward should be maximized at the lowest cost possible. This is evident in the analysis of this research work using the Linear and Algorithm solution pack called MATLAB - LINPROG, where optimum quantities were not only arrived at, but also with the possible cost reduction per unit/bag.

The surplus/slack units were also very interesting in this research work, since it creates opportunity for organizations to see the need for non-idleness of resources. To this end, organizations, having idea of the expected idle resources a priori, it can plan for optimum materials to be ordered for.

The Slack or Surplus column in a solution report tells you how close you are to satisfying a constraint as equality. This quantity, on less-than-or-equal-to (≤) constraints, is generally referred to as slack. On greater-than-or-equal-to (≥) constraints, this quantity is called a surplus.

If a constraint is exactly satisfied as equality, the slack or surplus value will be zero. If a constraint is violated, as in an infeasible solution, the slack or surplus value will be negative.

CONCLUSION

In the course of this research, a number of findings were deduced. These include the fact that managers of the organization of case study do not make use of modern scientific tools like LNPROG to plan their operations and production.

This optimization model can be used in a production environment to allow managers to choose the production schedule that maximizes profit. From the results of LINPROG software use in analyzing product mix of company six products (Broiler starter mash, Chicks mash, Growers mash, Broilers finisher mash, Layers mash and Supper layers), we are able to see the effects of changing resources on a profit and how it can help making informed decisions about resources used for production; maize, dawa (sorghum) , soya, fish meal, palm kernel cake, wheat offal, lime stone, bone meal, salt, pre-mix, methionine, lysine, bio-media, product demand, plant capacity and labour hours used. It also allows managers to make decisions regarding overtime for employees, temporary worker hiring, and raw materials usage and ordering. This model can be reused on an on-going basis providing management with the ability to do what-if scenarios and give them real insight into what effect changes in their resources might have on profitability.

Considering the challenges of today's world economy at the threshold of the 21st century and technological advancement and other competitive factors which interact in a complicated fashion to give rise to complex decision-making problem. It is not easy to make up schedule that is both realistic and economical. Hence, the acquisition of analytical method like linear programming and its software (LINPROG) for analyzing and manipulating large organization operations and production data gives edges to the firms.

REFERENCES

[1] Akingbade, J. F. (2002). *Operational research in theory and in practice: tools for rebirth and development of Nigeria*. University of Lagos Inaugural Series 2002: University of Lagos Press.

[2] Arogundade, K. K & Adebiyi, S. O (2011). *Operations and production management for management sciences, Vol. 2*. Ado-Ekiti: PETOA Educational Publisher.

[3] Albright, S. Christian & Winston, Wayne L. (2009). *Management science modeling, 3rd ed*. South-Western.

[4] Anderson, Sweeney & Williams (2003). *An introduction to management science – quantitative approaches to decisions making, 10th edition*. Thomson-South Western, pp. 31-84.

[5] Byrd, J. & Moore, L.T. (1978). Application of a Product Mix LP Model in Corporate Policy Making. *Management Science*, 24(13), 1342-1350.

[6] Caine, D. J. & Parker; B. J. (1996). Linear programming comes of age: decision support tool for every manager. *Management decision, Vol. 34, No.4*, pp. 46-53.

[7] Chung, S. H., Lee, A. H. I., and Pearn, W. L. (2005). Product mix optimization for semiconductor manufacturing based on AHP and ANP Analysis. *The Int. Journal of Advanced Manufacturing, Tech*., 2511-12, pp. 1144-1156.

[8] Dantzig & Thapa (1997). *Linear programming 1: introduction, springer series in operations research*. New York, USA: Springer-Verlag.

[9] Dantzig, G. (1963). *Linear programming & extensions*. The Rand- Princeton U.S.A. Press.

[10] Hillier, F. S. & Lieberman, Gerald J. (2005). *Operations Research; 7th edition*. Americas, New York: McGraw-Hill.

[11] Lee, T and Plenert, G (1996). Maximizing product mix Profitability-What's the best analysis tool. *Production Planning & Control*, 7(6), 547-553.

[12] Johnson, H. & Montgomery, P. J (1994). *Poultry: Freed and Nutrition, 2nd Edition*. Connecticut: Avi Publishing, Westport, 417-458.

[13] Rodin, R. (2000). Optimization in Operations Research. New Jersey U.S.A.: Prentice Hall.

[14] Savsar, M. & Aldaihani, M. (2011). Optimum Product Mix Selection from Waste Oil: A Linear Programming Application. *6th International Advanced Technologies Symposium* (IATS'11), 16-18 May 2011, Elazığ: Turkey.

[15] Sharma, J. K. (2008). *Operations research; theory and applications, 3rd edition*. India: Macmillan, Indian Ltd.

[16] Sulaimon O. A., Bilqis B. A., Ismail O. S. (2014), Linear Optimization Techniques for Product-Mix of Paints Production in Nigeria, Acta Universitatis Danubius. Œconomica, Vol 10, No 1, pp 1- 10.

[17] Winston, Wayne L. & Albright, S. Christian (1997). Practical Management Science. *Spread sheet modeling*. New York: Wadsworth Publishing Company.