

Illustration of Gauss - Seidel Method Using MATLAB

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Abstract- Considering the case of numerical algebra, the answer to system of linear equation is an assignment of ideals of variables such that entire equations in the systems are concurrently fulfilled. Gauss– Seidel method is an iterative method to answer a system of linear equations. It is also named as Liebmann method and this method is similar to the jacobbi method. Numerical solution of system of linear equation using gauss seidel method is given ahead.

Indexed Terms- System of linear equations, Gauss-Seidel Method, MATLAB solution, Theoretical solution, Applications.

I. INTRODUCTION

The direct methods of solving linear equations are recognized to have their difficulty. For example the problem with Gauss elimination approach lies in control of the accrual of rounding errors Turner. The Systems of linear equations ensue in bulky number of areas both directly in physical situations and ultimately in the numerical solutions of the other mathematics model. These applications happen in nearly all areas (i.e. physical, biological and social science). Linear systems are in the arithmetical solution of many problems (Such as: - optimization system of non-linear equations and partial differential equations). The most ordinary type of problem is to explain a square linear system is $[YX = Z]$ of moderate order with coefficient that are generally non zero, such linear system of any order are called dense since the coefficient matrix A is generally stored in the main memory of the computer in order to capably solve the linear system.

The problem of continuous mathematics are solved numerically with the help of algorithms which are created, analyzed and implemented by the numerical analysis which comes under the area of mathematics and computer science. These problems are originated

from real world applications (i.e. algebra, geometry and calculus) are involved of continuously varying variables. Direct methods or iterative methods are one of the two categories by which the solution of linear equations can be accomplished. In general we should prefer a direct method for the solution of liner equations. But, when it comes to matrices with a large number of zero elements we should use iterative methods which presents these elements are advantageous. Round off errors are controlled by the Gauss seidel method. In short gauss seidel method is the modification of Jacobi’s iteration method.

II. PROBLEM FORMULATION

Let us deem a system of ‘n’ linear equations with ‘n’ unknowns:-

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$$\begin{aligned}
 &y_{11}x_1 + y_{12}x_2 + y_{13}x_3 \dots\dots\dots + \\
 &y_{1n}x_n = z_1 \\
 &y_{21}x_1 + y_{22}x_2 + y_{23}x_3 \dots\dots\dots + \\
 &y_{2n}x_n = z_2
 \end{aligned}$$

In general,

$$\begin{aligned}
 &y_{n1}x_1 + y_{n2}x_2 + y_{n3}x_3 \dots\dots\dots + \\
 &y_{nn}x_n = z_n
 \end{aligned}$$

The matrix can also be represented as:-

In this the dioagonal element Y_{ij} doesn’t vanish. If so, the equation should be rearranged so as to satisfy the condition.

$$\begin{aligned}
 &YX=Z \text{—————(2)} \\
 &Y= \begin{matrix} y_{11} & y_{12} & y_{13} & \dots\dots\dots & y_{1n} \\ y_{21} & y_{22} & y_{23} & \dots\dots\dots & y_{2n} \end{matrix}
 \end{aligned}$$

In general,
 $y_{n1} \ y_{n2} \ y_{n3} \ \dots \ y_{nn}$
 $X = x_1$ And, $B = z_1$
 $x_2 \ z_2$

In general,
 $x_3 \ z_n$

So, the system of equation (1) can be rewritten as:
 $X_1 = z_1 - (y_{12}x_2 + y_{13}x_3 + \dots + y_{1n}x_n)$

Y11

$X_2 = z_2 - (y_{21}x_1 + y_{23}x_3 + \dots + y_{2n}x_n)$
 Y22

In general,

$X_1 = z_n - (y_{n1}x_1 + y_{n2}x_2 + \dots + y_{nn}x_n - 1)$
 Ynn

Now, if there was an availability of all the unknown guesses i.e. $X^0_1, X^0_2, X^0_3, \dots, X^0_n$. The value could be substituted in the equation (3) and the updated guess could be computed (i.e. $X^1_1, X^1_2, X^1_3, \dots, X^1_n$). There are several ways to accomplish this depending on the most recently computed guess.

III. ITERATIVE METHODS

These methods are used to get more specified accuracy in finding of roots for the system of linear equations. This phenomenon of sequence construction is known as iteration. Iterative methods starts with an initial approximation and then goes on to infinite sequence whose limit is the exact solution.

IV. GAUSS SEIDEL METHOD

Gauss seidel method is an iterative process used to solve system of n linear equations with x unknowns of a square matrix. The Gauss-Seidel Method permit the addict to manage round-off mistake. In fact, iterative methods can be used to recover the explanation we get by the direct methods. The majority of elimination methods are accountable to

bear on or after round-off errors. This method is alteration of the Jacobi's iteration method. It is defined on matrices with non-zero diagonals, but meeting is only sure if the matrix is either diagonally governing or symmetric positive definite. Also: If the physics of the difficulty are understood, a shut initial guess can be assumed, declining the number of iterations required.

V. MATLAB SOLUTION

Let us now find the roots of subsequent simultaneous equations by the Gauss-Seidel method.

$20x + y - 2z = 17$ ----- (1)
 $3x + 20y - z = -18$ ----- (1)
 $2x - 3y + 20z = 25$ ----- (1)

Result:-

Let's find the particular equation in the form as:

$A = (1/20) * (17 - y + 2z)$ ----- (2)
 $B = (1/20) * (-18 - 3x + z)$ ----- (2)
 $C = (1/20) * (25 - 2x + 3y)$ ----- (2)

We create approximation by $A_0 = B_0 = C_0 = 0$.

Substituting $B = B_0$ and $C = C_0$ in right hand side of first of equation (2)

$A_1 = 17/20 = 0.85$

In second of equation second put $A = A_1$ and $C = C_0$

$B_1 = (1/20) * (-18 - 3*0.85) = -1.0275$ Put $A = A_1$, $B = B_1$ in third of equation (2)

$C_1 = (1/20) * (25 - 2*0.85 + 3*-1.0275) = 1.011$

Likewise we get,

A2=1.002	B2=-0.9998	C2=0.9998
A3=1.0000	B3=-1.0000	C3=1.0000
A4=1.0000	B4=-1.0000	C4=1.0000

Now it is ample. If we watch over two sets of roots, they are approximately same.

Hence, roots of given instantaneous equations using Gauss-Seidel Method are:

CONCLUSION

A=1; B = -1; C=1

VI. THEORETICAL SOLUTION

Theory solution for:

$$20x+y-2z=17 \text{ ----- (1)}$$

$$3x+20y-z=-18 \text{ ----- (1)}$$

$$2x-3y+20z=25 \text{ ----- (1)}$$

Here, the equation is already arranged so, 1st iteration: (k=0)

$$X1^1 = (1/20)*[17-(1*0)-(-2*0)] = 0.85$$

$$X2^1 = (1/20)*[-18-(3*X1^1)-(-1*0)] = -1.0275$$

$$X3^1 = (1/20)*[25-(2*X1^1)-(-3*X2^1)] = 1.010875$$

2nd iteration: (k=1)

$$X1^2 = (1/20)*[17-(1*X2^1)-(-2* X3^1)] = 1.0024625$$

$$X2^2 = (1/20)*[-18-(3*X1^2)-(-1* X3^1)] = -0.999825625$$

$$X3^2 = (1/20)*[25-(2*X1^2)-(-3* X2^2)] = 0.999779906$$

Similarly,

3 rd iteration (k=2)	X13=0.99996 9271	X23=- 1.000006 395	X33= 1.000002 114
4 rd iteration (k=3)	X14=1.00000 0531	X24= - 0.999999 974	X34= 0.999999 95
5 rd iteration (k=4)	X15=0.99999 9993	X25= - 1.000000 001	X35= 1.000000 001
6 rd iteration (k=5)	X16=1.00000 000	X26= - 1.000000 00	X3600= 1.000000

VII. APPLICATIONS

The relevance of the Gauss–Seidel method is examine to obtain an iterative result of the system of thermal-radiation transfer equations for radiating, gripping and scattering media. Computation of an analysis setback are performed for the instance of the alteration shape of the nonlinear variant of the method for the finite-difference WDD scheme in planar geometry. The method is taken for the transport of γ -rays and neutrons.

Several methods can be used to solve system of linear equations some of them are direct methods and some are numerical method which are also termed as iterative method. In this review, two iterative methods for answering system of linear equations have been offered in which the Gauss-Seidel method proves to be actually fast. By the practical example, it was seen that answers were obtained within minute iteration.

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