Modal Analysis of Rear Coil Suspension System

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Abstract- In the vehicle suspension design, the optimal suspension should fulfill the following basic requirements: the ride comfort, reduction of dynamic road-tyre forces, and reduction of relative motions between the vehicle bodies. Controllable suspension systems are required to improve the compromise between the conflicting demands. Suspension system is the combination of various types of spring which include leaf springs, coil springs, air springs, rubber springs, torsion bar springs and shock absorber. They may be paired off in various combinations and attached by several different mounting techniques. Therefore, to check the comfort ability of a vehicle, it is necessary to find the deflections, stresses and stiffness or spring rates. Chrome Vanadium Steel ASTM-A231 (Cold Drawn) material is chosen to analyze for rear coil spring. Working frequencies of rear coil spring are 1.3324 Hz and 8.3027Hz respectively. Working frequencies do not match with natural frequency of rear coil spring at six mode shapes. Therefore, the designed coil spring is safe.

Indexed Terms- design; modal analysis; rear coil spring; suspension system;

I. INTRODUCTION

Suspension system is a very essential part of the automobile vehicle. Suspension system basically has two main components. Spring and shock absorber, both have their own function. Spring gives vertical motion to the wheel and provides allowance to work for shock absorber. Shock absorber absorbs the shock that may be transmitted to the body if not provided. When a car rides over bump, the springs are compressed, store the energy thus provide shock absorption. The energy will be released quickly when the springs bounce back. Dampers are employed to smooth and slow down the bounce motion, this is called "Damping". Without damper, the car will bounce up and down quickly, this is perceived as uncomfortable.

The fundamental purpose of ground vehicle suspension system is to maintain continuous contact between the wheels and road surface, and to isolate passengers or cargo from the vibration induced by the road irregularities. These two purposes are responsible for the handling quality and ride comfort respectively. Modal analysis of vehicle suspension system plays important role to improve the ride comfort. A great variety of spring materials are available to the designer, including plain carbon steels, alloy steels, and corrosion-resisting steels, as well as nonferrous materials such as phosphor bronze, spring brass, beryllium copper, and various nickel alloys. The UNS steels should be used in designing hot-worked, heavy-coil springs, as well as flat springs, leaf springs, and torsion bars.

II. THEORY FOR VIBRATION

Depending upon the cause, the vibration may be free or force. The free vibration may occur when the vehicle pass over an isolated irregularity in the road surface which may die of as a result of dissipation of energy in damping. On the other hand the forced vibration may result when disturbances occur persistently such as passing over obstacles on a proving road. In this case even if there may be damping, the vibration may persist and build up to an undesirable level.

A. Degrees of Freedom in Vibration

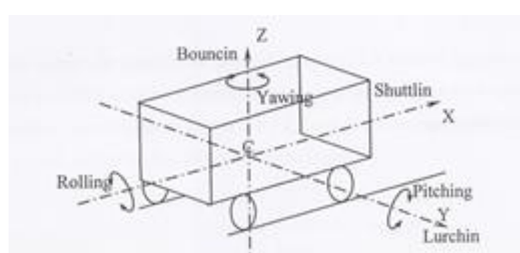

Figure 1. Quarter Car Suspension Model of 6 DOF

Figure 1 shows the six degree of freedom system on quarter car model.

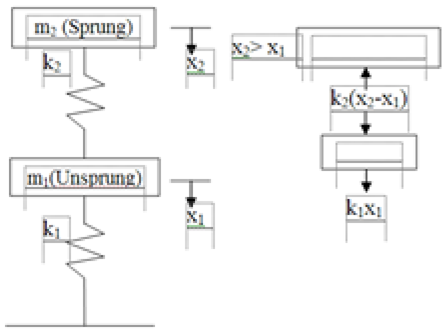

Figure 2. Quarter Car Model with 2DOF

For mass m1,

$$\sum F_y = ma = m_1\ddot{x}_1$$
$$k_2(x_2 - x_1) - k_1x_1 = m_1\ddot{x}_1$$
$$-m_1\ddot{x}_1 + k_2(x_2 - x_1) - k_1x_1 = 0$$
$$m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 = 0 \quad (1)$$

For mass m2,

$$\sum F_y = ma = m_2\ddot{x}_2$$
$$-k_2(x_2 - x_1) = m_2\ddot{x}_2$$
$$m_2\ddot{x}_2 + k_2(x_2 - x_1) = 0$$

$$m_2\ddot{x}_2 - k_2x_1 + k_2x_2 = 0 \quad (2)$$

Solving Equation (3.18) and (3.19) in matrix form,

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Let $M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$ and $K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$

Take $x = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}\cos\omega t$

$$\ddot{x} = \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} = -\omega^2\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}\cos\omega t$$

Then the equation (2) becomes,

$$-\omega^2[M]\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}\cos\omega t + [K]\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}\cos\omega t = 0$$

$$\{[K] - [M]\omega^2\}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}\cos\omega t = 0$$

$$\{[K] - [M]\omega^2\}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = 0 \quad (3)$$

$$\det([K] - [M]\omega^2) = 0$$

$$\det\begin{vmatrix} k_1 + k_2 - m_1\omega^2 & -k_2 \\ -k_2 & k_2 - m_2\omega^2 \end{vmatrix} = 0 \quad (4)$$

$$(k_1 + k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - {k_2}^2 = 0$$

Let $\lambda = \omega^2$

$$\omega_1 = \omega_{n,1} = \left(\frac{(m_2k_1 + m_2k_2 + m_1k_2)}{2m_2m_1} + \frac{\sqrt{(m_2k_1 + m_2k_2 + m_1k_2)^2 - 4m_2m_1k_2k_1}}{2m_2m_1}\right)^{1/2}$$

$$\omega_2 = \omega_{n,2} = \left(\frac{(m_2k_1 + m_2k_2 + m_1k_2)}{2m_2m_1} - \frac{\sqrt{(m_2k_1 + m_2k_2 + m_1k_2)^2 - 4m_2m_1k_2k_1}}{2m_2m_1}\right)^{1/2}$$

To find mode shape, the following equations are used.

$$\{[K] - [M]\omega^2\}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = 0$$

$$\begin{bmatrix} k_1 + k_2 - m_1\omega^2 & -k_2 \\ -k_2 & k_2 - m_2\omega^2 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = 0$$

$$(k_1 + k_2 - m_1\omega^2)X_1 - k_2X_2 = 0$$

$$\frac{X_2}{X_1} = \frac{(k_1 + k_2 - m_1\omega^2)}{k_2} \quad (5)$$

$$\frac{X_2}{X_1} = \frac{k_2}{(k_2 - m_2\omega^2)} \quad (6)$$

B. A Viscous Damped System Supported on a Base Excitation

The system considered is shown in Fig. The foundation is subject to harmonic vibration Y sinωt and required to determine the response, x, of the body.

$m\ddot{x} = \sum F$ (By using Newton's Method)

$$m\ddot{x} = -k(x-y) + c(\dot{x}-\dot{y})$$

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky \quad (7)$$

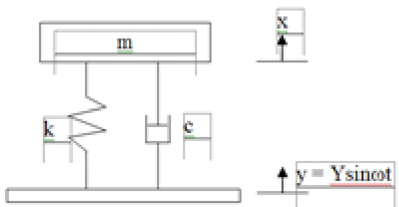

Fig. 3 . Single Degree of Freedom Modelof a Vibration System with Viscous Damping

Let $x = X\sin(\omega t - \emptyset)$

$$\dot{x} = \omega X\cos(\omega t - \emptyset)$$
$$\ddot{x} = -\omega^2 X\sin(\omega t - \emptyset)$$

Substituting into Eq (7)

Amplitude ratio $\left(\frac{X}{Y}\right) = \sqrt{\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}}$ (8)

Forcing frequency, $\omega = 2\pi f = 2\pi\frac{v}{\lambda}$ (9)

Damping factor, $\xi = \frac{c}{c_c}$ (10)

Frequency ratio $r = \frac{\omega}{\omega_n}$ (11)

Displacement $X = A\sin\omega t$ (12)

Velocity $\dot{X} = A\omega\cos\omega t$ (13)

Acceleration $\ddot{X} = -A\omega^2\sin\omega t = -\omega^2 X$ (14)

III. VIBRATION ANALYSIS FOR REAR COIL SPRING SUSPENSION WITH QUARTER CAR MODEL

Table 1 Specification data for Toyota Landcruiser Prado (KD-KZ195W)

Material	Chrome Vanadium Steel
Vehicle weight	1980 kg
Weight per person	90 kg
Vehicle capacity	6 person
Gross Weight	2520kg
FOS	1.8
Modulus of Rigidity	79289.70MPa
Density	7861.1kg/m^3
Young's Modulus	206.4GPa
Poisson's Ratio	0.29
Yield Strength	930.79MPa
Ultimate Strength	2068.42MPa
Wire diameter	16.92mm
Spring index	7.849
Number of active coil	5

From design calculation, Chrome vanadium steel is chosen to proceed the vibration analysis on static condition.

Unsprung Weight/ Vehicle weight = 0.15
Therefore Unsprung Weight/Mass m1= 1980×0.15
= 297 kg

Sprung Weight/mass, m_2 = 2520-297 = 2223 kg (Full Load)
m_2 =1980-297=1683kg (Empty load)

Unsprung Weight/Mass, m_1 = (297×0.6)/2=89.1kg
Sprung Weight/mass, m2 = (2520×0.6)/2=666.9kg
Force = 9.8×666.9 = 6535.62 N
Spring rate or Spring Stiffness = ks = k2 = W/y
y = $8FC^3n/dG$ = 109.67mm

$$k_s = k_2 = \frac{W}{y} = 64560\text{N/m}$$

Tyre stiffness $k_t = k_1 = 175540\text{N/m}$
The natural frequencies,

$$\omega_{n,1} = \left(\frac{(m_2k_1 + m_2k_2 + m_1k_2)}{2m_2m_1} - \frac{\sqrt{(m_2k_1 + m_2k_2 + m_1k_2)^2 - 4m_2m_1k_2k_1}}{2m_2m_1}\right)^{1/2}$$

$$\omega_{n,1} = 1.3324\text{Hz}$$

$$\omega_{n,2} = \left(\frac{(m_2k_1 + m_2k_2 + m_1k_2)}{2m_2m_1} + \frac{\sqrt{(m_2k_1 + m_2k_2 + m_1k_2)^2 - 4m_2m_1k_2k_1}}{2m_2m_1}\right)^{1/2}$$

$$\omega_{n,2} = 8.3027\text{Hz}$$

$$\lambda = 5\text{m}$$

At vehicle speed $v = 20\frac{\text{mile}}{\text{hr}} = 32.186\frac{\text{km}}{\text{hr}} = 8.94\text{m/s}$

Forcing frequency at $v = 8.94\text{m/s}$

$\omega = 2\pi\frac{v}{\lambda}$ = 1.7873 Hz

Frequency ratio for $\omega_{n,1}$, $r = \frac{\omega}{\omega_{n,1}} = \frac{1.7873}{1.3324} = 1.3414$

Frequency ratio for $\omega_{n,2}$,

$$r = \frac{\omega}{\omega_{n,2}} = \frac{1.7873}{8.3027} = 0.2152$$

Assume the amplitudes A_1, A_2, A_3 and A_4 as (0.04m, 0.08m, 0.12m and 0.16m)

At time period, $t = \frac{2\pi}{\omega}$

Displacement $X_1 = A_1\sin(\omega t) = 0.0043\text{m}$

$$X_2 = A_2\sin(\omega t) = 0.0087\text{m}$$
$$X_3 = A_3\sin(\omega t) = 0.0130\text{m}$$
$$X_4 = A_4\sin(\omega t) = 0.0173\text{m}$$

Velocity $\dot{X}_1 = A_1\omega\cos(\omega t) = 0.446\text{m/s}$

$$\dot{X}_3 = A_3\omega\cos(\omega t) = 1.33\text{m/s}$$
$$\dot{X}_4 = A_4\omega\cos(\omega t) = 1.78\text{m/s}$$

Acceleration $\ddot{X}_1 = -\omega^2X_1 = 0.54\text{m/s}^2$

$$\ddot{X}_2 = -\omega^2X_2 = 1.09\text{m/s}^2$$
$$\ddot{X}_3 = -\omega^2X_3 = 1.63\text{m/s}^2$$
$$\ddot{X}_4 = -\omega^2X_4 = 2.18\text{m/s}^2$$

At natural frequency $\omega_{n,1} = 8.37\text{rad/s}$

Damping coefficient,

$$c = \frac{F}{\dot{X}} = \frac{6535.62}{0.446} = 14653.85\text{Ns/m}$$

Critical damping coefficient,

$$c_c = 2m\omega_{n,1}$$
$$= 2 \times 666.9 \times 8.37$$
$$= 11163.91\text{Ns/m}$$

Damping factor $\xi = \frac{c}{c_c}$

$$\xi = \frac{14653.85}{11163.91}$$

$$\xi = 1.3$$

Amplitude ratio $\left(\frac{X}{Y}\right) = \sqrt{\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}}$

$$= \sqrt{\frac{1 + (2 \times 1.3 \times 1.3414)^2}{(1 - 1.3414^2)^2 + (2 \times 1.3 \times 1.3414)^2}}$$

= 1.0149

Table 2 Result data from vibration analysis of Chrome Vanadium steel

Spring stiffness	64560	N/m
Tyre stiffness	17550	N/m
Unsprung mass	89.1	kg
Sprung mass(Full load)	666.9	kg
Load applied or force	6535.62	N
Natural frequency 1	1.3324	Hz
Natural frequency 2	8.3027	Hz

At vehicle speed v = 8.94m/s, with wave length = 5m, the result data are as follows

Table 3 the result data of vibratory nature at v = 8.94m/s

Parameter	ω_{n1}	ω_{n2}	Unit
Forcing frequency	1.7873	1.7873	Hz
Damping coefficient	14653.85	14653.85	Ns/m

Critical damping coefficient	11163.91	69571	Ns/m
Frequency ratio	1.3414	0.2152	-
Damping factor	1.3	0.21	-
Amplitude ratio	1.015	1.048	-

At higher vehicle speed v = 17.88m/s, with same wave length, of the suspension system is changing as the result of different corresponding data values

Table 4 the result data of vibratory nature at v = 17.88m/s

Parameter	ω_{n1}	ω_{n2}	Unit
Forcing frequency	3.576	3.576	Hz
Damping coefficient	7343.39	7343.39	Ns/m
Critical damping coefficient	11163.91	69571	Ns/m
Frequency ratio	2.6838	0.4307	-
Damping factor	0.65	0.11	-
Amplitude ratio	0.5099	1.22	-

IV. RESULT AND DISSCUSSION

A modal analysis is typically used to determine the vibration characteristics (natural frequencies and mode shapes) of a structure or a machine component while it is being designed. It can also serve as a starting point for another, more detailed, dynamic analysis, such as a harmonic response or full transient dynamic analysis.

Modal analyses, while being one of the most basic dynamic analysis types available in ANSYS, can also be more computationally time consuming than a typical static analysis. In this research, the chromium vanadium steel is selected to design coil springs. Finally, a modal of coil spring was imported to ANSYS 17.0 and set up the boundary conditions for modal analysis. The important boundary for modal analysis is the fixed support. Fig.5.8 shows project schematic for modal analysis of the spring

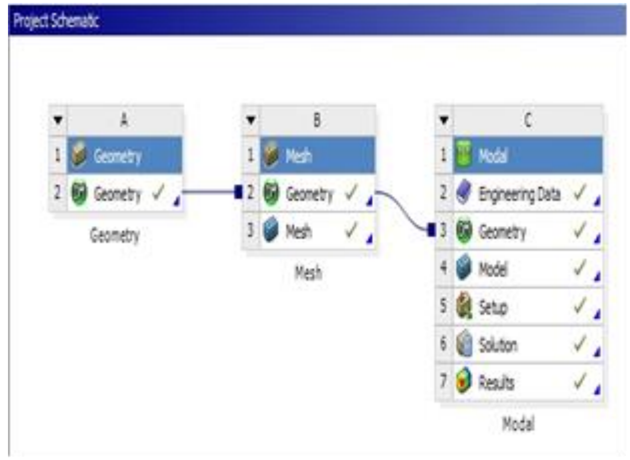

Fig 4. Schematic for Modal Analysis of Coil Spring

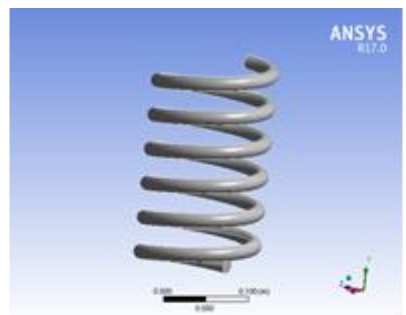

Fig 5. Geometry Model of Coil Spring for Modal Analysis

Fig.5 shows the model of coil spring imported from SolidWorks.

Mesh model of coil spring for modal analysis is illustrated as shown in Fig. 6.

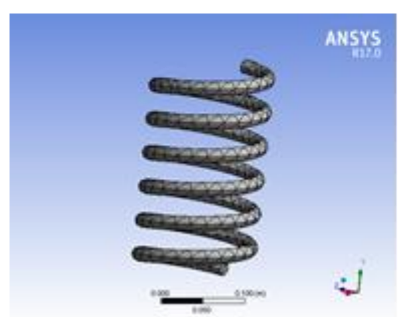

Fig 6. Meshed Model of Coil Spring for Modal Analysis

Modal analysis needs only boundary condition; it is not associated with the load apply because natural frequencies are resulted from free vibration. The boundary conditions are the same as in the case of static analytic.

One end of the spring is fixed as shown in Fig. 7 because this fixed point is attached to the chassis frame of the vehicle.

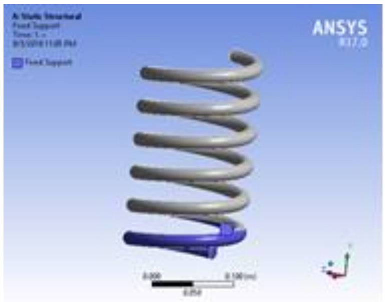

Fig 7 Boundary Condition of Coil Spring for Modal Analysis

The natural frequencies and mode shapes are important parameters in the design of a structure for dynamic loading conditions. The numerical results of natural frequencies for global mode shapes were compared with working frequency of coil spring.

There are corresponding mode shapes which describe the displacement of the system due to the vibration. The fundamental mode of vibration of a system is the mode having the lowest natural frequency. The shortest frequency is known as the natural frequency. Generally, the first mode of vibration is the one of primary interest.

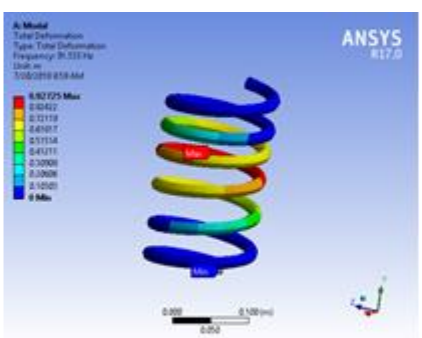

Fig 8. Natural Frequency of Mode Shape One for Coil Spring

Fig. 8 shows the natural frequency of mode shape one at total deformation for the spring. From the obtained results, the natural frequency of the designed spring at mode shape one is 91.53Hz. As the theoretical and simulated natural frequency, coil spring does not match so the spring has no tendency of resonance.

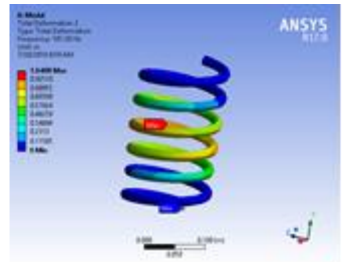

Fig 9. Natural Frequency of Mode Shape Two for Coil Spring

Fig. 9 shows the natural frequency of mode shape two at total deformation for the spring. From the obtained results, the natural frequency of the designed spring at mode shape two is 101.38Hz.

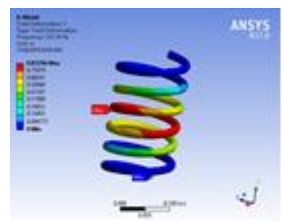

Fig 10. Natural Frequency of Mode Shape Three at for Coil Spring

In mode shape three, the corresponding working frequency is 123.14Hz at total deformation 85.29 mm as shown in Fig. 10

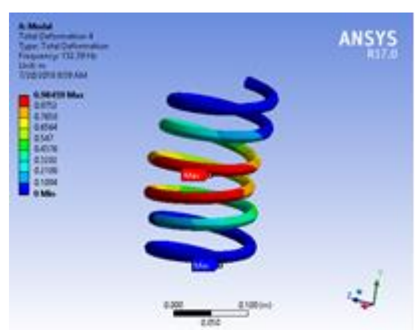

Fig 11. Natural Frequency of Mode Shape Four for Coil Spring

Fig 11 shows the natural frequency of mode shape four at total deformation. The corresponding working frequency is 132.39Hz at total deformation 98.45mm.

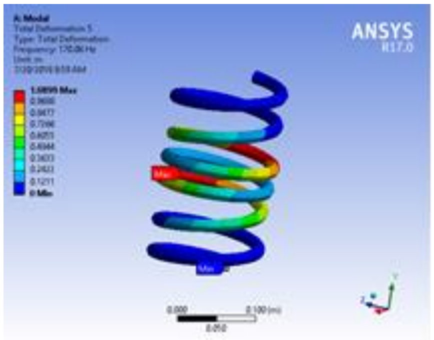

Fig 12 Natural Frequency of Mode Shape Five for Coil Spring

Fig. 12 shows the natural frequency of mode shape five at total deformation. From the result, the corresponding working frequency is 170.06Hz at total deformation 108.9mm.

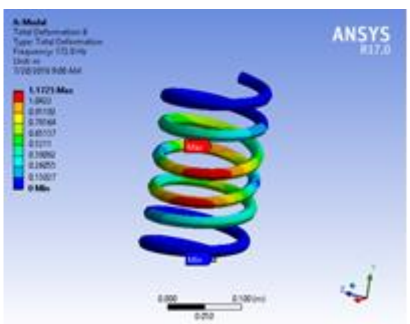

Fig 13. Natural Frequency of Mode Shape Six for Coil Spring

Fig. 13 shows the natural frequency of mode shape six at total deformation. From the result, the corresponding working frequency is 172.8Hz at total deformation 117.25mm.

Table 5.2, Natural Frequencies at Six Mode Shapes for Coil Spring.

The natural frequencies at six mode shapes of coil spring are 91.53, 101.38, 123.14, 132.39, 170.06 and 172.8 Hz which is with working frequencies of 1.3324Hz and 8.3027Hz. As a result, the natural frequencies at six global mode shapes for the spring have much higher value in compared of working frequencies. Therefore, the analysis is satisfied to perform on the actual work.

V. CONCLUSION

In this research work, rear coil spring design and modal analysis was considered. Chrome Vanadium steel is suitable for rear col spring. Rear coil spring was modeled by SolidWorks software and analyzed by ANSYS software. From the result it is observed that the working frequency of rear coil spring is 1.3324 and 8.3027 Hz. The natural frequencies at six mode shapes of coil spring are 91.53, 101.38, 123.14, 132.39, 170.06 and 172.8 Hz .Working frequencies do not match the natural frequencies of each mode shape. Therefore the design of rear coil suspension system is safe.

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