A Study on $e-$Domination of Cartesian Product of a Class of Path Semigraphs

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Abstract- In this paper, we study domination number of the cartesian product of some simple path semigraphs.

Index Terms- Cartesian Product, Domination number, Domination number, Path Semigraph, Semigraph.

I. INTRODUCTION

A semigraph S is a pair (V, X) where V is a nonempty set whose elements are called vertices of S and X is a set of ordered n-tuples $n \geq 2$ of distinct vertices called edges of S satisfying the following conditions :

i. any two edges have at most one vertex in common.

ii. two edges $E_1 = (u_1, u_2, ...u_m)$ and $E_2 = (v_1, v_2, ...v_n)$ are said to be equal iff

a. $m = n$ and

b. either $u_i = v_i$ or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$.

Semigraphs are introduced by E. Sampath kumar [9] in the year 2000, since then it has been an interesting field of research in graph theory. B. D. Acharya [1] discussed construction of semigraph from square matrices. B. Y. Bam and N. S. Bhave [2] have studied different types of degree sequences in semigraphs. S. P. Subbiah [10] have studied the relationship between topologies and discrete semigraphs.

There are different types of adjacency defined between two vertices in a semigraph. Two vertices u and v in a semigraph S are said to be $e-$adjacent if they are the end vertices of an edge. The basic concepts relating semigraphs are discussed in [4]. The properties of various types of adjacencies between vertices are deeply discussed in [5].

A subset D of V is said to be $e-$dominating set if for every $v \in V - D$ there exists an vertex $u \in D$ such that u and v are end vertices of an edge. The minimum cardinality of such a set D is called $e-$domination number of the semigraph S. It is denoted as $\gamma_e(S)$.

A path semigraph $S = (V, X)$ is a semigraph with the following properties.

i. it has no middle – end vertices.

ii. it has exactly two end vertices each with edge degree one.

iii. the edge degree of all other end vertices (if they exist) are exactly two.

Note that a path semigraph with no middle vertices is simply a path. The certain energies of path semigraphs have been studied in [6]. Charecteristic polynomial of path semigraphs have been studied in [7]. N. Murugesan and D. Narmatha [8] discussed $ca-$domination of Cartesian product of path semigraphs.

In this paper, we discuss the $e-$domination number of cartesian product of path semigraphs.

II. CARTESIAN PRODUCT OF PATH SEMIGRAPHS

Let $S_1 = (V_1, X_1)$ and $S_2 = (V_2, X_2)$ be two semigraphs. The Cartesian product of S_1 and S_2 denoted by $S_1 \square S_2$ is defined as follows :

Vertex set of $S_1 \square S_2$ is $V_1 \times V_2$ and the edge set is as given below:

i. For any vertex $u \in V_1$ and any edge $E = (v_1, v_2, \ldots, v_r)$ in X_2, $((u, v_1), (u, v_2), \ldots\ldots, (u, v_r))$ is an edge in $S_1 \square S_2$.

ii. Also, for any edge $E = (u_1, u_2, \ldots, u_s)$ in X_1 and for any vertex $v \in V_2$, $((u_1, v), (u_2, v), \ldots\ldots, (u_s, v))$ is an edge in $S_1 \square S_2$.

It has been discussed that cartesian product of two semigraphs is also a semigraph [3].

2.1 Example

Consider the following two semigraphs.

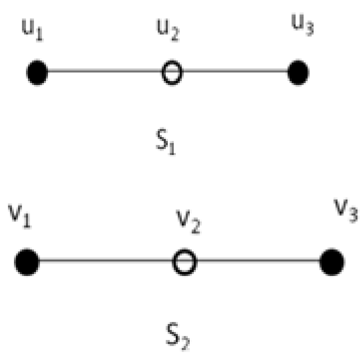

Fig. 2.1 two semigraphs S_1, S_2

The cartesian product of the above two semigraphs is given in fig. 2.2.

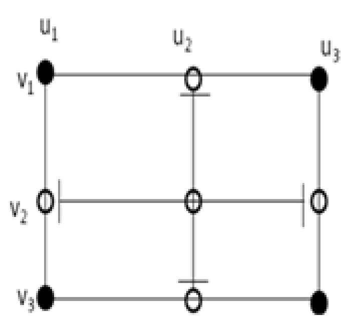

Fig. 2.2 cartesian product $S_1 \square S_2$

In this paper, we study the $e-$domination number of the cartesian product of simple path semigraphs. A simple path semigraph means, the path semigraph in which every edge contains exactly one middle vertex. A simple path semigraph with n edges is represented by $P_{s(n)}$. In this paper, we study $e-$domination number of $P_{s(n)} \square P_{s(m)}$, for $n = 1, 2, \ldots$ and $m = 1,2,3,\ldots,10$.

2.2 Theorem

$$\gamma_e [P_{s(n)} \square P_{s(1)}] = \begin{cases} 2n+2 & if \quad n = 3k;\; k = 1,2,3,\ldots \\ \frac{5}{3}(n-1)+6 & if \quad n = 3k+1;\, k = 2,3,\ldots \\ 2n+1 & if \quad n = 3k+2;\, k = 0,1,2,3,.. \end{cases}$$

Proof:

First let $k = 0$, then the cartesian products $P_{s(1)} \square P_{s(1)}$ and $P_{s(2)} \square P_{s(1)}$ are given in fig 2.3.

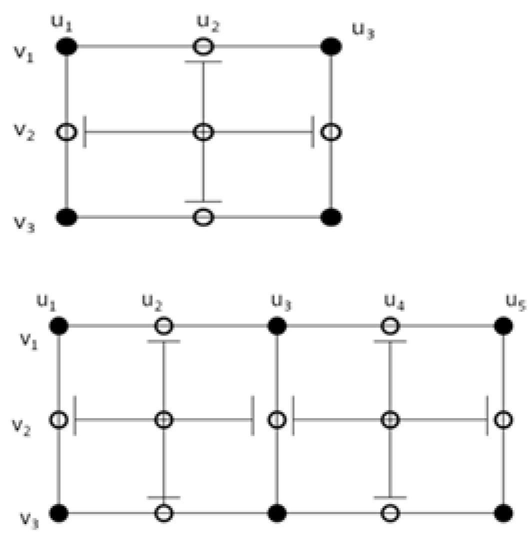

Fig. 2.3 $P_{s(1)} \square P_{s(1)}$; $P_{s(2)} \square P_{s(1)}$

Then the set of vertices $\{(u_2, v_1), (u_3, v_1), (u_3, v_2), (u_3, v_3)\}$ and $\{(u_2, v_1), (u_3, v_1), (u_4, v_1), (u_3, v_2), (u_3, v_3)\}$ form minimal $e-$dominating sets for $P_{s(1)} \square P_{s(1)}$ and $P_{s(2)} \square P_{s(1)}$ respectively. Hence, lemma follows for $k = 0$. Similarly we can easily verify the theorem for $k = 1$.

Next, let us take n = 3k, k = 2,3,4,….

The minimal $e-$dominating set D is

$$\left\{\begin{array}{l} \left\{\bigcup_{l=1}^{k-1}\left(u_{3(2l-1),}\ v_j\right) /\ j=1,2,3\right\} \bigcup \\ \left\{\begin{array}{l}\left(u_{6k-2}, v_1\right),\left(u_{6k-4}, v_1\right),\left(u_{6k-3}, v_1\right),\left(u_{6k-3}, v_2\right), \\ \left(u_{6k-3}, v_3\right),\left(u_{6k}, v_1\right),\left(u_{6k+1}, v_1\right),\left(u_{6k+1}, v_2\right)\end{array}\right\} \\ \bigcup_{l=1}^{3(k-1)}\left(u_{2l,}\ v_1\right) \end{array}\right\}$$

Now

$$\begin{aligned} |D| &= 3(k-1)+8+3(k-1) \\ &= 3k-3+8+3k-3 \\ &= 6k+2 \\ &= 6\left(\frac{n}{3}\right)+2 \\ &= 2n+2 \end{aligned}$$

Hence γ_e $[P_{s(n)} \square P_{s(1)}]$ = $2n+2$, when $n=3k; k=1,2,3,\ldots$

Let $n=3k+1; k=2,3,\ldots$. The minimal $e-$dominating set D is

$$\left\{\begin{array}{l} \left\{\bigcup_{l=1}^{k-1}\left(u_{(2l-1)3,}\ v_j\right);\ j=1,2,3\right\} \bigcup \\ \left\{\bigcup_{l=1}^{k-1}\left(u_{(2l-1)3-1}, v_1\right),\left(u_{(2l-1)3+1}, v_1\right)\right\} \bigcup \\ \left\{\left(u_{6(k-1),}\ v_1\right)\right\} \bigcup \\ \left\{\left(u_{6k-4}, v_1\right),\left(u_{6k-3}, v_1\right),\left(u_{6k-2}, v_1\right),\left(u_{6k-3}, v_2\right),\left(u_{6k-3}, v_3\right)\right\} \bigcup \\ \left\{\left(u_{6k}, v_1\right),\left(u_{6k+1}, v_1\right),\left(u_{6k+2}, v_1\right),\left(u_{6k+1}, v_2\right),\left(u_{6k+1}, v_3\right)\right\} \end{array}\right\}$$

$$\begin{aligned} |D| &= 3(k-1)+2(k-1)+11 \\ &= 3k-3+2k-2+11 \\ &= 5k+6 \\ &= 5\left(\frac{n-1}{3}\right)+6 \end{aligned}$$

Finally, let $n=3k+2, k=0,1,2,3,\ldots$. The corresponding minimal $e-$dominating set is

$$D=\left\{\begin{array}{l} \left\{\bigcup_{l=1}^{k+1}\left(u_{(2l-1)3,}\ v_j\right);\ j=1,2,3\right\} \bigcup \\ \left\{\bigcup_{l=1}^{k+1}\left(u_{(2l-1)3-1}, v_1\right),\left(u_{(2l-1)3+1}, v_1\right)\right\} \bigcup \\ \left\{\bigcup_{l=1}^{k}\left(u_{6l}, v_1\right)\right\} \end{array}\right\}$$

and

$$\begin{aligned} |D| &= 3\,(k+1)+2\,(k+1)+k \\ &= 6k+5 \\ &= 2n+1 \end{aligned}$$

Hence the theorem.

2.3 Theorem

$$\gamma_e\ [P_{s(n)} \square P_{s(2)}] = \begin{cases} 8\left(\frac{n}{3}\right)+3 & if\ \ n=3k \\ 8\left(\frac{n-1}{3}\right)+5 & if\ \ n=3k+1 \\ 8\left(\frac{n-2}{3}\right)+7 & if\ \ n=3k+2 \end{cases}$$

$k=0,1,2,\ldots..$

Proof :

The cartesian product $P_{s(1)} \square P_{s(2)}$ and $P_{s(2)} \square P_{s(2)}$ is given in the following fig.

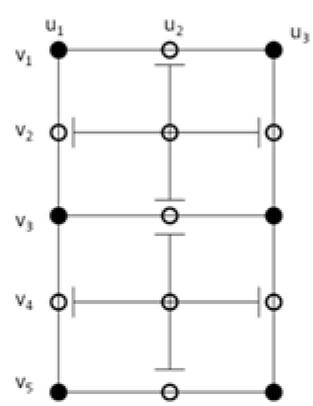

Fig. 2.4 $P_{s(1)} \square P_{s(2)}$

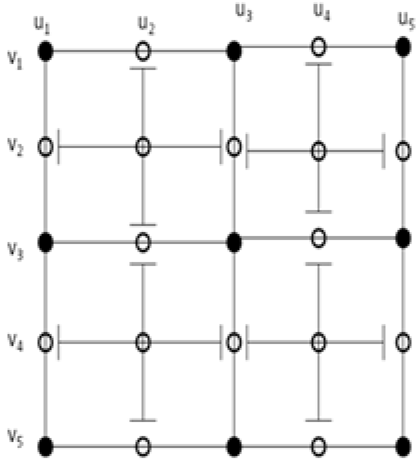

Fig. 2.5; $P_{s(2)} \square P_{s(2)}$

First let us assume $k = 0$. The set of vertices $\{(u_3,v_2),(u_3,v_3),(u_3,v_4),(u_1,v_3),(u_2,v_3)\}$ and $\{(u_3,v_1),(u_3,v_2),(u_3,v_3),(u_3,v_4),(u_3,v_5),(u_2,v_3),(u_4,v_3)\}$ form a minimal $e-$dominating sets for $P_{s(1)} \square P_{s(2)}$; $P_{s(2)} \square P_{s(2)}$ respectively. Hence the lemma is true for $k = 0$. Similarly we can verify the theorem for $k = 1$.

Next, let us take $n = 3k, k = 2,3,\ldots$The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k}\left(u_{3(2l-1)}, v_j\right) / \ j = 1,2,3,4,5.$$
$$\bigcup_{l=1}^{3k}\left(u_{2l}, v_3\right)$$
$$\bigcup\left(u_{6k+1}, v_j\right) / \ j = 2,3,4.$$

Now

$$|D| = 5k + 3k + 3$$
$$= 8k + 3$$
$$= 8\left(\frac{n}{3}\right) + 3$$

Hence $\gamma_e\ [P_{s(n)} \square P_{s(2)}] = 8\left(\frac{n}{3}\right) + 3$, when $n = 3k, k = 2,3,\ldots$.

Let us take $n = 3k+1,\ k = 2,3,\ldots$The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k}\left(u_{3(2l-1)}, v_j\right) / \ j = 1,2,3,4,5.$$
$$\bigcup_{l=1}^{3k+1}\left(u_{2l}, v_3\right)$$
$$\bigcup\left(u_{6k+1}, v_j\right) / \ j = 2,3,4.$$

$$|D| = 5k + (3k+1) + 1 + 3$$
$$= 8k + 5$$
$$= 8\left(\frac{n-1}{3}\right) + 5$$

Finally, let us take $n = 3k+2,\ k = 2,3,\ldots$

The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k+1}\left(u_{3(2l-1)}, v_j\right) / \ j = 1,2,3,4,5.$$
$$\bigcup_{l=0}^{3k+1}\left(u_{2l+2}, v_3\right)$$

Now

$$|D| = 5(k+1) + (3k+2)$$
$$= 8k + 7$$
$$= 8\left(\frac{n-2}{3}\right) + 7$$

2.4 Theorem

$$\gamma_e\ [P_{s(n)} \square P_{s(3)}] = \begin{cases} \frac{13(n)}{3} + 5 & if \quad n = 3k \\ \frac{13(n-1)}{3} + 8 & if \quad n = 3k+1 \\ \frac{13(n-2)}{3} + 11 & if \quad n = 3k+2 \\ k = 0,1,2,\ldots & \end{cases}$$

Proof :

First let us assume $k = 0$. The set of vertices $\{(u_3,v_2),(u_3,v_3),(u_3,v_4),(u_1,v_3),(u_2,v_3),(u_3,v_6),(u_2,v_7),(u_3,v_7)\}$

and

$$\left\{\begin{matrix}(u_3,v_1),(u_3,v_2),(u_3,v_3),(u_3,v_4),(u_3,v_5),(u_4,v_5),\\(u_2,v_3),(u_4,v_3),(u_2,v_5),(u_3,v_6),(u_3,v_7)\end{matrix}\right\}$$

form a minimal $e-$dominating sets for $P_{s(1)} \square P_{s(3)}$; $P_{s(2)} \square P_{s(3)}$ respectively. Hence the lemma is true for $k=0$. Similarly we can verify the theorem for $k=1$.

Next, let us take $n=3k, k=2,3,\ldots.$

The minimal $e-$dominating set is

$$D=\bigcup_{l=1}^{k}\left(u_{3(2l-1)},v_j\right)/\ j=1,2,\ldots\ldots,7$$
$$\bigcup_{l=1}^{3k}\left(u_{2l},v_j\right)/\ j=3,5$$
$$\bigcup\left(u_{6k+1},v_j\right)/\ j=2,3,4,5,6.$$

Now

$$\begin{aligned}|D|&=7k+6k+5\\&=13k+5\\&=13\left(\frac{n}{3}\right)+5\end{aligned}$$

Hence $\gamma_e\ [P_{s(n)} \square P_{s(3)}]=\ 13\left(\frac{n}{3}\right)+5$, when $n=3k, k=2,3,\ldots..$

Let us take $n=3k+1,\ k=2,3,\ldots.$

The minimal $e-$dominating set is

$$D=\bigcup_{l=1}^{k}\left(u_{3(2l-1)},v_j\right)/\ j=1,2,\ldots\ldots,7.$$
$$\bigcup_{l=1}^{3k+1}\left(u_{2l},v_j\right)/\ j=1,5.$$
$$\bigcup\left(u_{6k+1},v_5\right)$$
$$\bigcup\left(u_{6k+3},v_j\right)/\ j=1,2,4,5,6.$$

Now

$$\begin{aligned}|D|&=7k+(6k+2)+1+5\\&=13k+8\\&=13\left(\frac{n-1}{3}\right)+8\end{aligned}$$

Finally, let us take $n=3k+2,\ k=2,3,\ldots.$

The minimal $e-$dominating set is

$$D=\bigcup_{l=1}^{k+1}\left(u_{3(2l-1)},v_j\right)/\ j=1,2,\ldots.,7.$$
$$\bigcup_{l=1}^{3k+2}\left(u_{2l},v_j\right)/\ j=3,5.$$

Now

$$\begin{aligned}|D|&=(7k+7)+(6k+4)\\&=13k+11\\&=13\left(\frac{n-2}{3}\right)+11\end{aligned}$$

Hence the theorem.

2.5 Theorem

$$\gamma_e\ [P_{s(n)} \square P_{s(4)}]=\begin{cases}15\left(\frac{n}{3}\right)+6 & if\quad n=3k\\15\left(\frac{n-1}{3}\right)+10 & if\quad n=3k+1\\15\left(\frac{n-2}{3}\right)+13 & if\quad n=3k+2\\k=0,1,2,\ldots\end{cases}$$

Proof :

First let us assume $k=0$. The set of vertices $\{(u_3,v_2),(u_3,v_3),(u_3,v_4),(u_1,v_3),(u_2,v_3),(u_3,v_6),(u_1,v_7),($ and $\{(u_3,v_j)/\ j=1,2,\ldots.,9\}$ and $\{(u_i,v_j)/\ i=2,4\ ,\ \ j=3,7\}$ form a minimal $e-$dominating sets for $P_{s(1)} \square P_{s(4)}$; $P_{s(2)} \square P_{s(4)}$ respectively. Hence the lemma is true for $k=0$.

Next, let us take $n = 3k, k = 1,2,3,\ldots.$

The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k} \left(u_{3(2l-1)}, v_j\right) / \ j = 1,2,\ldots\ldots,9.$$
$$\bigcup_{l=1}^{3k} \left(u_{2l}, v_j\right) / \ j = 3,7$$
$$\bigcup \left(u_{6k+1}, v_j\right) / \ j = 2,3,4,6,7,8.$$

Now

$$|D| = 9k + 6k + 6$$
$$= 15k + 6$$
$$= 15\left(\frac{n}{3}\right) + 6$$

Hence γ_e $[P_{s(n)} \square P_{s(4)}] = 15\left(\frac{n}{3}\right) + 6$, when $n = 3k, k = 1,2,3,\ldots..$

Let us take $n = 3k + 1,\ k = 1,2,3,\ldots.$

The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k} \left(u_{3(2l-1)}, v_j\right) / \ j = 1,2,\ldots\ldots,9.$$
$$\bigcup_{l=1}^{3k+1} \left(u_{2l}, v_j\right) / \ j = 3,7.$$
$$\bigcup \left(u_{6k+1}, v_j\right) / \ j = 2,3,4,6,7,8.$$
$$\bigcup \left(u_{6k+3}, v_j\right) / \ j = 3,7.$$

Now

$$|D| = 9k + (6k + 2) + 6 + 2$$
$$= 15k + 10$$
$$= 15\left(\frac{n-1}{3}\right) + 10$$

Finally, let us take $n = 3k + 2,\ k = 1,2,3,\ldots.$

The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k+1} \left(u_{3(2l-1)}, v_j\right) / \ j = 1,2,\ldots.,9.$$
$$\bigcup_{l=1}^{3k+2} \left(u_{2l}, v_j\right) / \ j = 3,7.$$

Now

$$|D| = 9(k+1) + (6k + 4)$$
$$= 15k + 13$$
$$= 15\left(\frac{n-2}{3}\right) + 13$$

Hence the theorem.

2.6 Theorem

$$\gamma_e\ [P_{s(n)} \square P_{s(5)}] = \begin{cases} \frac{17(n)}{3} + 7 & if \quad n = 3k \\ \frac{17(n-1)}{3} + 11 & if \quad n = 3k+1 \\ \frac{17(n-2)}{3} + 15 & if \quad n = 3k+2 \\ k = 0,1,2,\ldots & \end{cases}$$

Proof :

First let us assume $k = 0$. Here $\left\{\left(u_3, v_j\right) / \ j = 2,3,4,6,8,9,10\right\}$ and $\left\{\left(u_3, v_j\right) / \ j = 1,2,\ldots.,9\right\}$ and $\left\{(u_1, v_3), (u_2, v_3), (u_1, v_9), (u_2, v_9)\right\}$ form a minimal $e-$dominating sets for $P_{s(1)} \square P_{s(5)}$. And $\left\{\left(u_3, v_j\right) / \ j = 2,3,4,6,8,9,10\right\}$ $\left\{\left(u_i, v_j\right) / i = 1,2,3,4,5,\ \ j = 3,9\right\}$ form a minimal $e-$dominating sets for $P_{s(2)} \square$ $P_{s(5)}$ respectively. Hence the lemma is true for $k = 0$. Similarly we can verify the theorem for $k = 1$.

Next, let us take $n = 3k, k = 2,3,\ldots.$

The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k}\left(u_i, v_{3(2l-1)}\right) / i = 1,2,\ldots\ldots,6k+1.$$
$$\bigcup_{l=1}^{k}\left(u_{3(2l-1)}, v_j\right) / j = 2,4,6,8,10.$$
$$\bigcup\left(u_{6k+1}, v_j\right) / j = 2,4,6,8,10.$$

Now

$$|D| = 2(6k+1)+5k+5$$
$$= 17k+7$$
$$= 17\left(\frac{n}{3}\right)+7$$

Let us take $n = 3k+1,\ k = 2,3,\ldots.$

The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k}\left(u_i, v_{3(2l-1)}\right) / i = 1,2,\ldots\ldots,6k+3.$$
$$\bigcup_{l=1}^{k+1}\left(u_{3(2l-1)}, v_j\right) / j = 2,4,6,8,10.$$

Now

$$|D| = 2(6k+3)+5k+5$$
$$= 17k+11$$
$$= 17\left(\frac{n-1}{3}\right)+11$$

Finally, let us take $n = 3k+2,\ k = 2,3,\ldots.$

The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k}\left(u_i, v_{3(2l-1)}\right) / i = 1,2,\ldots.,6k+5.$$
$$\bigcup_{l=1}^{k+1}\left(u_{3(2l-1)}, v_j\right) / j = 2,4,6,8,10.$$

Now

$$|D| = 2\ (6k+5)+5k+5$$
$$= 17k+15$$
$$= 17\left(\frac{n-2}{3}\right)+15$$

Hence the theorem.

2.7 Theorem

$$\gamma_e\ [P_{s(n)} \square P_{s(6)}] = \begin{cases} \frac{22(n)}{3}+9 & if \quad n = 3k \\ \frac{22(n-1)}{3}+14 & if \quad n = 3k+1 \\ \frac{22(n-2)}{3}+19 & if \quad n = 3k+2 \\ k = 0,1,2,\ldots \end{cases}$$

Proof :

First let us assume $k = 0$. The set of vertices $\{(u_3, v_j) / j = 2,4,6,8,10,12,13\}$, $\{(u_i, v_j) / i = 1,2,3,\ j = 3,9\}$ and (u_2, v_{13}) form a minimal $e-$dominating sets for $P_{s(1)} \square P_{s(6)}$. $\{(u_3, v_j) / j = 2,4,6,8,10,12,13\}$ and $\{(u_i, v_j) / i = 1,2,3,4,5,\ j = 3,9\}$, (u_4, v_{13}) form a minimal $e-$dominating sets for $P_{s(2)} \square P_{s(6)}$ respectively. Hence the lemma is true for $k = 0$. Similarly we can verify the theorem for $k = 1$.

Next, let us take $n = 3k, k = 2,3,\ldots.$

The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k}\left(u_i, v_{3(2l-1)}\right) / i = 1,2,\ldots\ldots,6k+1.$$
$$\bigcup_{l=1}^{k}\left(u_{3(2l-1)}, v_j\right) / j = 2,4,6,8,10,12,13.$$
$$\bigcup_{l=1}^{3k}\left(u_{2l}, v_{13}\right)$$
$$\bigcup\left(u_{6k+1}, v_j\right) / j = 2,4,6,8,10,12,13.$$

Now

$$|D| = 2(6k+1)+7(k+1)+3k$$
$$= 22k+9$$
$$= 22\left(\frac{n}{3}\right)+9$$

Let us take $n = 3k+1,\ k = 2,3,\ldots.$

The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k}\left(u_i, v_{3(2l-1)}\right)/\ i = 1,2,\ldots\ldots,6k+3.$$
$$\bigcup_{l=1}^{k+1}\left(u_{3(2l-1)}, v_j\right)/\ j = 2,4,6,8,10,12,13.$$
$$\bigcup_{l=1}^{3k+1}\left(u_{2l}, v_{13}\right)$$

Now

$$|D| = 2(6k+3)+7(k+1)+3k+1$$
$$= 22k+14$$
$$= 22\left(\frac{n-1}{3}\right)+14$$

Finally, let us take $n = 3k+2,\ k = 2,3,\ldots.$

The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k}\left(u_i, v_{3(2l-1)}\right)/\ i = 1,2,\ldots.,6k+5.$$
$$\bigcup_{l=1}^{k+1}\left(u_{3(2l-1)}, v_j\right)/\ j = 2,4,6,8,10,1213.$$
$$\bigcup_{l=1}^{3k+2}\left(u_{2l}, v_{13}\right)$$

Now

$$|D| = 2(6k+5)+7(k+1)+3k+2$$
$$= 22k+19$$
$$= 22\left(\frac{n-2}{3}\right)+19$$

2.8 Theorem

$$\gamma_e\ [P_{s(n)} \square P_{s(7)}] = \begin{cases} 24\left(\frac{n}{3}\right)+10 & if\quad n = 3k \\ 24\left(\frac{n-1}{3}\right)+16 & if\quad n = 3k+1 \\ 24\left(\frac{n-2}{3}\right)+21 & if\quad n = 3k+2 \\ k = 0,1,2,\ldots \end{cases}$$

Proof :

First let us assume $k = 0$. The set of vertices $\{(u_3, v_j)/\ j = 2,4,6,8,10,12,13,14,15\}$ and $\{(u_1,v_3),(u_2,v_3),(u_1,v_9),(u_2,v_9),(u_2,v_{15})\}$ (u_2, v_{13}) form a minimal $e-$dominating sets for $P_{s(1)} \square P_{s(7)}$. $\{(u_3, v_j)/\ j = 2,4,6,8,10,12,13,14,15\}$ and $\{(u_i, v_j)/\ i = 1,2,3,4,5,\ \ j = 3,9\}$, (u_2, v_{15}), (u_4, v_{15}) form a minimal $e-$dominating sets for $P_{s(2)} \square P_{s(7)}$ respectively. Hence the lemma is true for $k = 0$. Similarly we can verify the theorem for $k = 1$.

Next, let us take $n = 3k, k = 2,3,\ldots.$ The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k}\left(u_i, v_{3(2l-1)}\right)/\ i = 1,2,\ldots\ldots,6k+1.$$
$$\bigcup_{l=1}^{k}\left(u_{3(2l-1)}, v_j\right)/\ j = 2,4,6,8,10,12,13,14,15.$$
$$\bigcup_{l=1}^{3k}\left(u_{2l}, v_{15}\right)$$
$$\bigcup\left(u_{6k+1}, v_j\right)/\ j = 2,4,6,8,10,12,13,15.$$

Now

$$|D| = 2(6k+1)+9k+3k+8$$
$$= 24k+10$$
$$= 24\left(\frac{n}{3}\right)+10$$

Let us take $n = 3k+1,\ k = 2,3,\ldots.$

The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k}\left(u_i, v_{3(2l-1)}\right) / i = 1,2,\ldots\ldots,6k+3.$$
$$\bigcup_{l=1}^{k+1}\left(u_{3(2l-1)}, v_j\right) / j = 2,4,6,8,10,12,13,14,15.$$
$$\bigcup_{l=1}^{3k+1}\left(u_{2l}, v_{15}\right)$$

Now

$$|D| = 2(6k+3) + 9(k+1) + 3k + 1$$
$$= 24k + 16$$
$$= 24\left(\frac{n-1}{3}\right) + 16$$

Finally, let us take $n = 3k+2,\ k = 2,3,\ldots.$

The minimal $e-$dominating set is

$$D = \bigcup_{l=1}^{k}\left(u_i, v_{3(2l-1)}\right) / i = 1,2,\ldots,6k+5.$$
$$\bigcup_{l=1}^{k+1}\left(u_{3(2l-1)}, v_j\right) / j = 2,4,6,8,10,1213,14,15.$$
$$\bigcup_{l=1}^{3k+2}\left(u_{2l}, v_{15}\right)$$

Now

$$|D| = 2(6k+5) + 9(k+1) + 3k + 2$$
$$= 24k + 21$$
$$= 24\left(\frac{n-2}{3}\right) + 21$$

Hence the theorem.

As we proved the above theorems, we can also prove the following theorems.

2.9 Theorem

i. $\gamma_e[P_{s(n)} \square P_{s(8)}] =$

$$\begin{cases} 26\left(\frac{n}{3}\right) + 11 & if \quad n = 3k \\ 26\left(\frac{n-1}{3}\right) + 17 & if \quad n = 3k+1 \\ 26\left(\frac{n-2}{3}\right) + 23 & if \quad n = 3k+2 \\ k = 0,1,2,\ldots \end{cases}$$

ii. $\gamma_e[P_{s(n)} \square P_{(9)}] =$

$$\begin{cases} 31\left(\frac{n}{3}\right) + 13 & if \quad n = 3k \\ 31\left(\frac{n-1}{3}\right) + 20 & if \quad n = 3k+1 \\ 31\left(\frac{n-2}{3}\right) + 27 & if \quad n = 3k+2 \\ k = 0,1,2,\ldots \end{cases}$$

iii. $\gamma_e[P_{s(n)} \square P_{(10)}] =$

$$\begin{cases} 33\left(\frac{n}{3}\right) + 14 & if \quad n = 3k \\ 33\left(\frac{n-1}{3}\right) + 22 & if \quad n = 3k+1 \\ 33\left(\frac{n-2}{3}\right) + 29 & if \quad n = 3k+2 \\ k = 0,1,2,\ldots \end{cases}$$

VI. CONCLUSION

It has been interesting fact to study $e-$domination number of $P_{s(n)} \square P_{s(m)}$. In this paper, it has been discussed the $e-$domination number of cartesian product graphs $P_{s(n)} \square P_{s(m)}$, m=1,2,…,10 but for all values of n.

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