Unsteady Laminar Forced Convections in A Pipe Having Sector As Its Cross Section

DR.SARVESH NIGAM

Assistant Professor, Department of Mathematics, Pt. J. L. N. P. G. College Banda (U.P.)

Abstract- The purpose of this paper is to analyze the problem of forced convections in liquid flows through sufficiently long straight channel of sector as its cross section. The results are obtained in terms of tabulated Lommel, Bessel and associated functions.

The Hankels and sine transform are made use of. In the present work we imagine the following:

a) The flow is laminar and unsteady and liquid properties are constant.
b) Heat source is present in the channel.
c) Flow is undeveloped (both Hydro dynamically and thermally).
d) The liquid and wall temperature increase or decrease linearly at the same rate in the direction of flow.
e) The axis of the channel and flow direction are in the positive direction of Z-axis.

I. INTRODUCTION

Heat transfer problems of forced convections in channels have contributed an attractive and useful subject of investigation for several years. It may however be said that

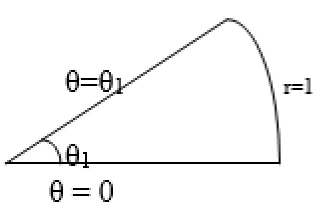

Figure: SECTOR

the laminar forced convection problems of channels is one of the most fundamental and important problems in heat transfer as it forms the basis of the investigation of several other problems of heat transfer.

Only the cases of round conduits have been investigated in detail by several research workers for unsteady flow. The cases of non-circular ducts have not been investigated.

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(a) The flow is laminar and unsteady and liquid properties are constant.
(b) Heat source is present in the channel.
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(e) The axis of the channel and flow direction are in the positive direction of z-axis.

II. FORMULATION OF THE PROBLEM

The governing equations after Tao (1) are

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial z} + \frac{1}{\nu}\frac{\partial u}{\partial t} \quad \ldots\ldots\ldots (1)$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{\ell\Delta}{k}\frac{\partial T}{\partial z} u - \frac{Q}{k} + \frac{1}{k}\frac{\partial T}{\partial t} \quad \ldots\ldots\ldots (2)$$

Where Q is the heat source intensity, k is thermal conductivity, μ is coefficient of viscosity, ℓ is the density, u is local velocity in axial direction, T is modified temperature. $T' - T_w$ and T' is local temperature and T_w is wall temperature which remains uniform throughout the wall.

Let us further assume

$C_1 (t) = \frac{1}{\mu}\frac{\partial p}{\partial z}$

$C_2 = \frac{\partial T}{\partial z}\frac{\ell\Delta}{k}$

$C_3 (t) = \frac{Q}{K}$

So equation (1) and (2) becomes

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = C_1 (t) + \frac{1}{\nu}\frac{\partial u}{\partial t} \text{……… (3)}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = C_2 u - C_3 (t) + \frac{1}{k}\frac{\partial T}{\partial t} \text{……… (4)}$$

III. TRANSFORMATION

Let us suppose that

$x = r \cos\theta$

$y = r \sin\theta$

Now on transforming in polar co-ordinates equation (3) and (4) transforms to

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = C1 (t) + \frac{1}{\nu}\frac{\partial u}{\partial t} \text{……… (5)}$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = C2u - C_3 (t) + \frac{1}{K}\frac{\partial T}{\partial t} \text{……… (6)}$$

IV. BOUNDARY CONDITIONS

U = 0	T = 0	for r=1
		θ=0, θ=α t ≥ 0
U = 0	T = 0	r≤1 for t=0
		0< θ≤ α nd t= ∞

V. SOLUTION OF THE PROBLEM

Multiplying equation (5) and (6) by e^{-st} and integrating with the limits 0 to ∞ and putting

$$u_L = \int_0^\infty e^{-st} u \, dt$$

$$\frac{\partial^2 u_L}{\partial r^2} + \frac{1}{r}\frac{\partial u_L}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_L}{\partial \theta^2} = C1 (t) + \frac{s}{\nu} u_L \text{……… (7)}$$

Where

$C_1(t) = \int \infty \; e^{-st} C_1(t)dt$……….(8)

Now multiplying (7) by sinþθ and integrating within the limits 0 to α and using the boundary conditions we get

$$\frac{\partial^2 \bar{u}_L}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{u}_L}{\partial r} - \frac{p^2 \bar{u}_L}{r^2} = \frac{2\, C1}{þ} (t) + \frac{s}{\nu} \bar{u}_L \text{……… (9)}$$

If we write

$$\bar{u}_L = \int_0^\infty u_L \sin þ\theta d\theta$$

where þ's are the roots of the equation

Þα = (2n+1) π……….. (10)

Now multiplying equation (9) by $rJ_þ(qr)$ where $J_þ(qr)$ are Bessel function of the first kind and q's are the roots of the equation

$J_þ(q) = 0$…………(11)

And integrating within the limits 0 to 1we get by Snedden(2)

$$-q^2 \bar{u}_{LH} = \frac{2}{Þ} \overline{C_1 (t)} \lambda + \frac{s}{\nu} \bar{u}LH \text{ ………… (12)}$$

Where $\bar{u}_{LH}$ is Hankel's transform of $\bar{u}_L$ and

$$\lambda = \int_0^1 rJ_þ (qr)$$

$$= \frac{1}{q^2} [þqJ_þ (q) S_{0,þ-1}(q) - qJ_{þ-1} (q) S_{1,þ} (q) + þ] \text{…………… (13)}$$

Where $S_{\mu,\nu}$ is the Lommel function .Similarly on undergoing the same process we get from equation (6)we get

$$-q^2 \bar{T}_{LH} = C_2 \bar{u}_{LH} - \frac{2}{Þ} \lambda \, C_3 \overline{(t)} + \frac{s}{k} T\bar{L}H \text{ ………… (14)}$$

Where

$$\overline{C_3(t)} = \int_0^\infty e^{-st} C_3(t)dt \text{ …………(15)}$$

From equation (12) we get

$$\bar{u}_{LH} = \frac{-2 \lambda \nu \overline{C_1 (t)}}{Þ (s+ \nu q^2)} \text{ ………….. (16)}$$

on substituting the value of $\bar{u}_{LH}$ in equation (15) we get

$$\overline{T_{LH}} = \frac{2\lambda K}{(s+kq^2)}\left[\overline{C_3(t)} - \frac{\nu C_2\, \overline{C_1(t)}}{þ\,(s+\nu q^2)}\right] \quad \text{............ (17)}$$

Now by the inversion formula 6.47 of Trainter (3)

$$\bar{u}_L = \frac{-4\nu\, C_1(t)}{Þ} \sum_q \frac{\lambda\, J_þ(qr)}{(s+\nu q^2)\; J^2_{þ+1}(q)} \quad \text{.............. (18)}$$

and by Sine inversion theorem we have

$$u_L = -\frac{8\nu\, \overline{C_1(t)}}{\alpha} \sum_q \sum_þ \frac{\lambda S_þ(qr)\sin þ\,\theta}{(s+\nu q^2)\, þ J2_{þ+1}(q)} \quad \text{............. (19)}$$

And

$$T_L = \sum_q \sum_þ \frac{8\lambda k}{\alpha(s+kq^2)\,þ}\left\{\overline{C3(t)} - \frac{\nu C_2\, \overline{C_1(t)}}{(s+\nu q^2)}\right\}\frac{Jþ(qr)\sin þ\,\theta}{J^2_{þ+1}(q)} \quad \text{........... (20)}$$

VI. PARTICULAR CASE

Let $C_1(t) = C_1 e^{-rt}$

$C_3(t) = C_3\, e^{-\beta t}$

$$\overline{C_1(t)} = \int_0^\infty C_1 e^{-rt}\, C^{-st}\, dt$$

So

$$= \frac{C_1}{s+r}$$

$$C_3(t) = \frac{C_3}{s+\beta}$$

And

On substituting the values of $C_1(t)$ and $C_3(t)$ we get

$$u_L = -\frac{8\nu\, C_1}{\alpha} \sum_q \sum_þ \frac{\lambda J_þ(qr)\sin þ\,\theta}{(s+r)(s+\nu q^2)\, þ J2_{þ+1}(q)} \quad \text{.............. (21)}$$

$$T_L = \sum_q \sum_þ \frac{8\lambda k}{\alpha(s+kq^2)\,þ}\left\{\frac{C3(t)}{(s+\beta)} - \frac{\nu C_2 C_1}{(s+\nu q^2)(s+r)}\right\}\frac{Jþ(qr)\sin þ\,\theta}{J^2_{þ+1}(q)} \quad \text{............ (22)}$$

by Laplace inverse transform

$$u = -\frac{8\nu\, C_1}{\alpha} \sum_q \sum_þ \frac{\lambda J_þ(qr)\sin þ\,\theta}{þ J^2_{þ+1}(q)\,(\nu q^2 - r)}\left[e^{-rt} - e^{-\nu q2t}\right] \quad \text{.............. (23)}$$

$$T = \sum_q \sum_þ \frac{8\lambda k}{\alpha þ}\left[\frac{C_3(e^{-\beta t} - e^{-kq2t})}{(kq2-\beta)} - \nu C_1 C_2 \left\{\frac{e^{-kq2t}}{q^2(\nu-k)(r-kq^2)} + \frac{e^{-\nu q2t}}{q^2(k-\nu)(r-\nu q^2)} + \frac{e^{-rt}}{(kq^2-r)(\nu q2-r)}\right\}\right]\frac{J_þ(qr)\sin þ\,\theta}{J^2_{þ+1}(q)} \quad \text{.....(24)}$$

The mean velocity u_m is given by

$$u_m = \frac{1}{A}\int_D u\, dA$$

$$= -\frac{8\nu\, C_1}{\alpha} \sum_q \sum_þ \frac{\lambda\left[e^{-rt} - e^{-\nu q2t}\right]}{þ J^2_{þ+1}(q)\,(\nu q2-r)}\; \frac{\int_0^1\int_0^\alpha r\, J_þ(qr)\sin þ\,\theta\, dr d\theta}{\int_0^1\int_0^\alpha r\, dr d\theta}$$

$$= -\frac{32 C_1 \nu}{\alpha^2} \sum_q \sum_þ \frac{\lambda^2 \left(e^{-rt} - e^{-\nu q2t}\right)}{þ^2\,(\nu q2-r)\, J^2_{þ+1}(q)} \quad \text{............ (25)}$$

Where λ is given by the equation (13), where mean temperature is given by

$$T_m = \frac{1}{A}\int_D T\grave{}\, dA$$

$$= \sum_q \sum_þ \frac{8\lambda k}{\alpha þ}\left[\frac{C_3(e^{-\beta t} - e^{-kq2t})}{(kq^2-\beta)} - \nu C_1 C_2 \left\{\frac{e^{-kq2t}}{q^2(\nu-k)(r-kq^2)} + \frac{e^{-\nu q2t}}{q^2(k-\nu)(r-\nu q^2)} + \frac{e^{-rt}}{(kq^2-r)(\nu q^2-r)}\right\}\right]\frac{\int_0^1\int_0^\alpha r J_þ(qr)\sin þ\theta\, dr d\theta}{J^2_{þ+1}(q)\int_0^1\int_0^\alpha r dr d\theta}$$

$$T_m = \sum_q \sum_þ \frac{32\lambda^2 k}{\alpha^2 þ^2 J^2_{þ+1}(q)}\left[\frac{C_3(e^{-\beta t} - e^{-kq2t})}{(kq^2-\beta)} - \nu C_1 C_2 \left\{\frac{e^{-kq2t}}{q^2(\nu-k)(r-kq^2)} + \frac{e^{-\nu q2t}}{q^2(k-\nu)(r-\nu q^2)} + \frac{e^{-rt}}{(kq^2-r)(\nu q^2-r)}\right\}\right] \quad \text{... (26)}$$

Mixed mean temperature T_M is given by

$$T_M = \frac{1}{Au_m}\int_D u\, T dA$$

$$\int_D uT dA = -\int_0^1\int_0^\alpha \sum_q \sum_þ \frac{8\nu C_1}{\alpha}\frac{\lambda J_þ(qr)\sin þ\theta\,(e^{-rt} - e^{-\nu q2t})}{þ J^2_{þ+1}(q)(\nu q^2-r)} \cdot \sum_q \sum_þ \frac{8\lambda k}{\alpha þ}\left[\frac{C_3(e^{-\beta t} - e^{-kq2t})}{(kq^2-\beta)} - \nu C_1 C_2\left\{\frac{e^{-kq2t}}{q^2(\nu-k)(r-kq^2)} + \frac{e^{-\nu q2t}}{q^2(k-\nu)(r-\nu q^2)} + \right.\right.$$

$$\frac{e^{-rt}}{(kq^2-r)(vq2-r)}\}]\ \frac{J_þ(qr)\sin þ\theta\, r dr d\theta}{J^2_{þ+1}(q)}$$

Now

$$\int_0^{\alpha} \sin þ\theta . \sin þ`\theta = \frac{\alpha}{2} \quad \text{If} \quad þ = þ`$$

$$= 0 \quad \text{If} \quad þ \neq þ`$$

By equation (48) of page 70 of Erdelyl (4)

$$\int_0^1 r \underline{J_þ(q_m r)\, J_þ(q_n r)}\, dr = 0 \qquad \text{if } m \neq n$$

$$= [J_{þ+1}(q_n)]^2 \qquad \text{if } m=n$$

So we get

$$\int_D uTdA = -32\sum_q\sum_þ \frac{\lambda^2 vk}{\alpha þ^2 J^2_{þ+1}(q)}\ C_1[e^{-rt}-e^{-vq2t}].\ [C_3\frac{(e^{-\beta t}-e^{-kq2t})}{(kq^2-\beta)} - vC_1C_2\{\frac{e^{-kq2t}}{q^2(v-k)(r-kq^2)} + \frac{e^{-vq2t}}{k^2(k-v)(r-vq^2)} + \frac{e^{-rt}}{(kq^2-r)(vq^2-r)}\}]$$

So

$$T_m = \frac{2\sum_q\sum_þ \frac{\lambda^2 vk}{\alpha þ^2 J^2_{þ+1}(q)}\ C_1[e^{-rt}-e^{-vq2t}].\ [\frac{C_3(e^{-\beta t}-e^{-kq2t})}{(kq^2-\beta)} - rC_1C_2\{\frac{e^{-kq2t}}{q^2(v-k)(r-kq^2)} + \frac{e^{-vq2t}}{q^2(k-v)(r-vq^2)} + \frac{e^{-rt}}{(kq^2-r)(vq^2-r)}\}]}{\sum\sum \frac{C_1 v}{\alpha}\frac{\lambda 2(e^{-rt}-e^{-vq2t})}{þ^2(r-vq^2)J^2_{þ+1}(q)}} \quad \ldots\ldots\ldots\ldots (27)$$

The overall heat transfer rate is given by

$$\bar{q} = [C_2 u_m - C_3(t)]\, kA$$

$$\bar{q} = [\frac{32\, C1vC2}{\alpha^2}\sum_q\sum_þ \frac{\lambda^2(e^{-rt}-e^{-vq2t})}{þ^2(r-vq^2)J^2_{þ+1}(q)} - C_3 e^{-\beta t}]\, kA$$

$$= [\frac{16\, v}{A^2\alpha}\sum_q\sum_þ \frac{(e^{-rt}-e^{-vq2t})\lambda^2}{þ^2(r-vq^2)J^2_{þ+1}(q)} - \frac{C_3 e^{-\beta t}}{AC_1C_2}]\, kA2\, C_1C_2 \ldots\ldots\ldots\ldots (28)$$

Let $C_1C_2 = C_4$

$$\frac{C_3}{C_1C_2A} = C_5$$

So $$q` = [\frac{16\, v}{A^2\alpha}\sum_q\sum_þ \frac{(e^{-rt}-e^{-vq2t})\lambda^2}{þ^2(r-vq^2)J^2_{þ+1}(q)} - C_5 e^{-\beta t}] \quad \ldots\ldots\ldots\ldots (29)$$

Where q` is dimensionless heat transfer rate

The heat transfer coefficient h based on mixed mean temperature is given by

$$h = -\frac{\bar{q}}{ST_M}$$

$$= \frac{-[\frac{16\, v}{\alpha}\sum_q\sum_þ \frac{(e^{-rt}-e^{-vq2t})\lambda^2}{þ^2(r-vq^2)J^2_{þ+1}(q)} - A^2C_5e^{-\beta t]}[\sum_q\sum_þ \frac{v\lambda^2}{\alpha þ^2}\frac{(e^{-rt}-e^{-vq2t})}{(r-vq^2)J^2_{þ+1}(q)}]}{2[2+\alpha][\sum_q\sum_þ \frac{\lambda^2 v(e^{-rt}-e^{-vq2t})}{\alpha þ^2 J^2_{þ+1}(q)}.[\frac{C_5(e^{-\beta t}-e^{-kq2t})}{kq^2\beta} - v\{\frac{e^{-kq2t}}{q^2(v-k)(r-kq^2)} + \frac{e^{-vq2t}}{q^2(k-v)(r-vq^2)} + \frac{e^{-rt}}{(kq^2-r)(vq2-r)}\}]} \quad \ldots (30)$$

Nusselts number based on the mixed temperature are given by

$$N_u = \frac{hD_e}{K}$$

Where D_e is equivalent hydraulic diameter.

i.e. $$D_e = \frac{4A}{S} = \frac{4.1/2\alpha}{[2+\alpha]}$$

$$= \frac{2\alpha}{2+\alpha}$$

So N_u is given by

$$= \frac{-\alpha}{(2+\alpha)^2 k} \cdot \frac{\left[\frac{16\nu}{\alpha}\sum_q\sum_p \frac{(e^{-rt}-e^{-\nu q^2 t})\lambda^2}{p^2(r-\nu q^2)\,J^2_{p+1}(q)} - \frac{\alpha^2 C_5}{4} e^{-\beta t}\left[\sum_q\sum_p \frac{\nu\lambda^2(e^{-rt}-e^{-\nu q^2 t})}{\alpha p^2(r-\nu q^2)\,J^2_{p+1}(q)}\right]\right.}{\left[\sum_q\sum_p \frac{\lambda^2 \nu (e^{-rt}-e^{-\nu q^2})}{\alpha p^2 J^2_{p+1}(q)} \cdot \left[\frac{C_5(e^{-\beta t}-e^{-kq^2 t})}{kq2-\beta}\right] - \nu\left\{\frac{e^{-kq^2 t}}{q^2(\nu-k)(r-kq^2)} + \frac{e^{-\nu q^2 t}}{q^2(k-\nu)(r-\nu q^2)} + \frac{e^{-rt}}{(kq^2-r)(\nu q2-r)}\right]\right.} \quad \text{.......... (31)}$$

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