Analysis of Fault Detection Algorithm for Power System Transmission Lines Using DFT and FFT

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Abstract- the Development of Fault Detection Algorithm for power system transmission line consists of two algorithms. Discrete Fourier Transform (DFT) and Fast Fourier Transform (FFT). A block diagram is developed which shows the developmental stages of the algorithms. The DFT and FFT detection algorithms are developed based on their individual applications of Fourier Transform (FT) discretization of phase values of voltage and current to obtain the discrete versions (V_P and I_P) as the required data for the research. The discretization is followed by DFT and FFT applications to the output result (V_P and I_P) of the discretization to obtain the time – domain and frequency – domain components. Mathematical approach of the implementation of the algorithm is performed and the values of the phase discrete voltage and current values, pre-fault and three phase fault voltage and current magnitudes for DFT and FFT applications were obtained and tabulated.

Indexed Terms- DFT, FFT, Algorithm, Voltage, Current, impedance, Transmission Line, Faults, Matlab, Simulink

I. INTRODUCTION

There has been rapid growth of the power grids all over the world in the past decades. This growth has led to the installation of large number of new transmission and distribution lines to ensure provision of sufficient power supply to end user.

Moreover, the continuity of this power supply is highly important, this is because, and insufficient power supply can cause a lot of problems or set - back to the country. The problem can be summarized as decrease in economy and infrastructural development of that country.

One of the factors which can cause discontinuity of this power supply from reaching the end users is faults (Balance or Unbalanced faults).

Therefore, the fault developed or occurred on the power system transmission line must be cleared as fast as possible to enable the isolated part of the transmission line due to fault receive power supply and to prevent further damage of the power system equipment or cause black – out in the country.

Various methods have been employed for detecting, classifying, locating and analyzing these faults. They include; Impedance measurement based method Travelling wave based method Signal processing based method (Discrete Fourier Transform (DFT), Fast Fourier Transform (FFT), Wavelet (WT) and S-Transform (ST) Artificial Intelligent system (AI) based method etc.

II. METHODOLOGY

Methodological procedure followed in this research is shown on figure 1. The Modeling of the power system transmission line shown in figure 2 is done using Matlab/Simulink 2016a and Onitsha transmission station parameters, such as Real Power 87.7MW, Reactive Power 21.40MVAR, and Active Power of 90.27MVA with frequency of 50Hz and line distance of 96km from Onitsha to Enugu. The modeled transmission line is shown on figure 4.

- Mathematical Approach for DFT: The mathematical expressions for DFT is employed using the sampled three phase voltage and current signals (v (t) and i (t)) as inputs of the DFT algorithm. In this mathematical approach, three phase sets of samples or data of v(t) and i(t) each are used with number of samples N equals to three (3). Two types of samples are used, the pre-fault and faulted samples. Table 1 and 2 shows both type of samples assumed to have been obtained through equations 2.1 to 2.6.

Va = Vpa sin (Θ ± Ø) 2.1
Vb = Vpb sin (Θ ± Ø) 2.2
Vc = Vpc sin (Θ ± Ø) 2.3
Ia = Ipa sin (Θ ± ψ) 2.4
Ib = Ipb sin (Θ ± ψ) 2.5
Ic = Ipc sin (Θ ± ψ) 2.6

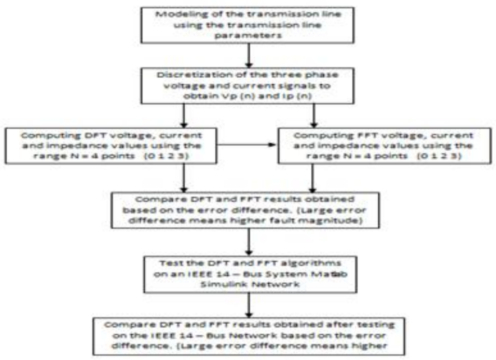

Figure 1: Block diagram of the procedure

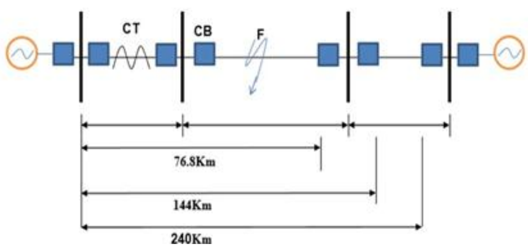

Figure 2: 330/132KV transmission line representing Onitsha – Enugu

$$v_p(n) = \frac{2}{3}[\, v_a + v_b(n)e^{\frac{j2\pi}{3}} + v_c(n)e^{\frac{-j2\pi}{3}}\,]$$
2.7

$$i_p(n) = \frac{2}{3}[\, i_a + i_b(n)e^{\frac{j2\pi}{3}} + i(n)e^{\frac{-j2\pi}{3}}\,]$$
2.8

$$v(t) = V_p \sin(\theta + \emptyset)$$
2.9

$$i(t) = I_p \sin(\theta + \varphi)$$
2.10

$$V_x(n) = \sum_{n=0}^{N-1} v(t)e^{\frac{-j2\pi n}{N}}$$
2.11

$$I_x(n) = \sum_{n=0}^{N-1} i(t)e^{\frac{-j2\pi n}{N}}$$
2.12

Equations 2.7 and 2.8 are phase voltage and current discretizing equations respectively. Equation 2.9 and 2.10 is used to obtain the actual voltage and current complex or composite signals respectively. However equations 2.11 and 2.12 are used to compute the DFT voltage and current values.

The DFT of a signal v(t) or i(t) is given as Equations 2.11 and 2.12 respectively. Where, Vx(n) or Ix(n) is the DFT of the sequence v(n), or I(n) and using v(n) and I(n) equal to;

V(n) = $[v(0), v(1), v(2), v(3)]^T$ and I(n) = $[I(0), I(1), I(2), I(3)]^T$ respectively for 4 – point DFT application having

N = 4 (number of DFT samples).
V (n) can be represented in matrix form as follows:

$$[(v(0)\ v(1)\ v(2)\ v(3)]^T = \begin{bmatrix} 1 & 1 & 1 & 1 & \ldots 1 \\ 1 & W & W2 & W3 & \ldots WN\text{-}1 \\ 1 & W2 & W4 & W6 & \ldots WN\text{-}2 \\ 1 & WN\text{-}1 & WN\text{-}2 & WN\text{-}3 & \ldots W \end{bmatrix} [(v(0)\ v(1)\ v(2)\ v(3)]^T$$

Where
W = exp ((-J2π)/ (N)) and W = W^{2N} = 1 2.13
For a 4 – point DFT application, where N = 4

Thus,
W = exp ((-J2π)/ (N)) = exp ((-J2π)/ (4))
= exp ((-Jπ)/ (2)) = a 2.14
a2 = -j, 2.15
a3 = -ja = -a* 2.16
a4 = -1 2.17

Therefore, the DFT of the voltage and current signals for any condition is given by equation 2.18

$$V(k) = \sum_{0}^{3} v(n)e^{\frac{-j\pi nk}{2}} = \sum_{0}^{3} v(n)(-j)^{nk}$$
2.18

$$I(k) = \sum_{0}^{3} i(n)e^{\frac{-j\pi nk}{2}} = \sum_{0}^{3} i(n)(-j)^{nk}$$
2.19

Thus,

$$[(v(0)\ v(1)\ v(2)\ v(3)]^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} [(v(0)\ v(1)\ v(2)\ v(3)]^T$$

Then the DFT output values can be used to plot the time dependent graph (time – Domain) and spectrum graph (frequency – domain) which shows the magnitude or amplitude of the signal relationship with frequency (Nimrod Peleg, 2002).

- Mathematical Approach for FFT:

FFT is highly efficient computer algorithm for estimating DFT and has been developed since Mid-60s. It relies on the fact that the standard DFT involves a lot of redundant calculations. It is given as;

$$V(k) = \sum_{k=0}^{N-1} v(n) e^{\frac{-j2\pi nk}{N}} = \sum_{k=0}^{N-1} v(n) W_N^{nk}$$

2.20

$$W_N^{nk} = e^{\frac{-j2\pi nk}{N}}$$

2.21

It can be computed several times as computation proceeds using butterfly method of flow.

Firstly, the integer product nk repeats for different combinations of k and n. Secondly, is a periodic function with only N distinct value (N = 4 as in previous section). The FFT is simplest if N is an integer power of 2 such as 4 or 8 etc. (Nimrod Peleg, 2002).

The equations butterfly method is given by;

$V(0) = [v(0) + v(2)] + W_4^0 [v(1) + (v(3)]$ 2.22

$V(1) = [v(0) + v(2)] + W_4^1 [v(1) + (v(3)]$ 2.23

$V(2) = [v(0) + v(2)] + W_4^0 [v(1) + (v(3)]$ 2.24

$V(3) = [v(0) + v(2)] + W_4^1 [v(1) + (v(3)]$ 2.25

Where, $[V(0)\ V(1)\ V(2)\ V(3)]^T$ is the output results of FFT. The DFT output results for both pre-fault and fault conditions were applied as inputs to the FFT mathematical expression. After the computation of the FFT for both pre-fault and faulty conditions, the results are shown in chapter four.

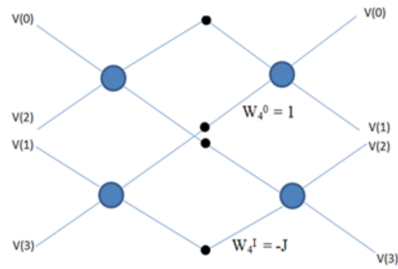

Figure 5: Butterfly flow process for FFT 4 – Point Application

III. SIMULATION AND RESULT ANALYSIS

The per unit voltage and current values of table 3.1 and 3.2 are magnitude or amplitude values which whensampled into the equations 2.7 and 2.8 respectively will be used to obtain the discrete values tabulated in table 3.3 and 3.4.

S/N	$v_p(n)$	$i_p(n)$	$z_p(n)$	FAULTS
1	2.7591	0.6121	4.5076	L-G
2	0.9331	0.2999	3.1114	LL-G
3	2.4220	0.0000	0.0000	L-L
4	0.8932	0.3393	2.6295	LLL

Table 3.3: Pre-fault discrete voltage, currentand impedance values

S/N	$v_p(n)$	$i_p(n)$	$z_p(n)$	FAULTS
1	2.7425	1.1076	2.5488	L-G
2	2.0997	3.8590	0.5441	LL-G
3	3.0976	0.5488	5.6443	L-L
4	1.1598	1.5992	0.7253	LLL

Table 3.4: Three phase fault discrete voltage, current and impedance values

Table 3.3, 3.4 illustrate the discrete values (V_p and I_p) obtained for pre-fault and fault conditions under four categories of fault. One symmetrical and three unsymmetrical faults.

The DFT is applied to these discrete values following the methodological procedure and using equation 2.18 and 2.19 to determine the time – domain and frequency – domain components of voltage and current which contain the information such as fault information about the transmission line.

S/N	N - POINTS	$V_{0,1,2,3}$ for N = 4	$I_{0,1,2,3}$ for N = 4	V(n)	I(n)	Z(n)	FAULTS
1	0	0.6316	0.2399	-0.9998	-0.3503	2.8531	NIL
2	1	-0.0125	-0.0047	13.6095	0.5902	27.102	NIL
3	2	-0.7266	-0.2465	0.8103	0.3371	2.4037	NIL
4	3	-0.8924	-0.3390	13.6095	0.5902	27.102	NIL

Table 3.5: Mathematical Approach DFT Application for Pre-fault Condition at N – POINTS(k = 0… N – 1)

S/N	N - POINTS	$V_{0,1,2,3}$ for N = 4	$I_{0,1,2,3}$ for N = 4	V(n)	I(n)	Z(n)	FAULTS
1	0	0.8201	1.1308	-1.1974	-1.6512	0.7252	LLL
2	1	-0.0162	-0.0223	2.0175	2.7819	0.7252	LLL
3	2	-0.8427	-1.1619	1.1524	1.5891	0.7252	LLL
4	3	-1.1588	-1.5978	2.0175	2.7819	0.7252	LLL

Table 3.6: Mathematical Approach DFT Application for Three Phase Fault Condition at N – POINTS (k = 0… N – 1)

S/N	N - POINTS	$DFTV_{0,1,2,3}$ for N = 4	$DFTI_{0,1,2,3}$ for N = 4	FFT V(k)	FFT I(k)	FFT Z(k)	FAULTS
1	0	0.6316	0.2399	-0.9999	1.1672	-0.8566	LLL
2	1	-0.0125	-0.0047	1.6183	0.6874	2.3542	LLL
3	2	-0.7266	-0.2465	0.9999	-1.1936	-0.8377	LLL
4	3	-0.8924	-0.3390	-1.6183	0.6874	-2.3542	LLL

Table 3.7: Mathematical Approach FFT Application for Pre-fault Condition at N – POINTS (k = 0… N – 1)

S/N	N - POINTS	$DFTV_{0,1,2,3}$ for N = 4	$DFTI_{0,1,2,3}$ for N = 4	FFT V(k)	FFT I(k)	FFT Z(k)	FAULTS
1	0	0.8201	1.1308	-1.1976	-1.9512	0.6138	LLL
2	1	-0.0162	-0.0223	1.6628	2.2927	0.7253	LLL
3	2	-0.8427	-1.1619	1.1524	-1.6066	-0.7173	LLL
4	3	-1.1588	-1.5978	1.6628	2.2927	0.7253	LLL

Table 3.8: Mathematical Approach FFT Applicationfor Three Phase Fault Condition at N – POINTS(k = 0…, N – 1)

The DFT and FFT results obtained using mathematical approach are tabulated on table 3.5 & 3.6 and table 3.7 & 3.8 for three phase (LLL) pre-fault and fault conditions respectively.

To obtain the time – domain and frequency – domain waveform and spectrum diagram representing V(n), I(n) and V(k), I(k) respectively, we plot V(n) and I(n) individually against time for time domain and against the frequency for frequency domain (spectrum waveform) using Matlab.

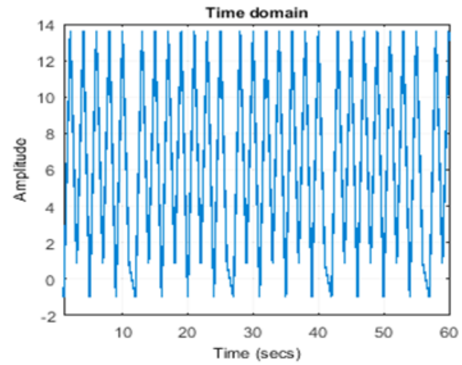

Figure 3.1: Mathematical Approach Pre-Fault DFT Voltage (Vn) t - domain Signal Waveform

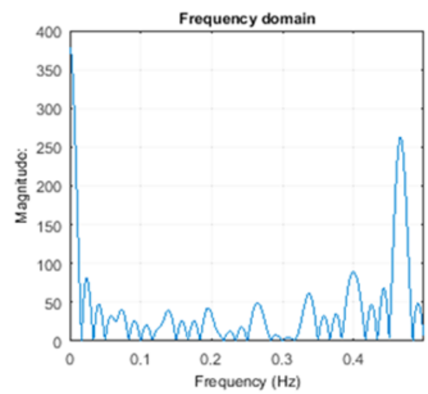

Figure 3.2: Mathematical Approach Pre-Fault DFT Voltage (Vn) s – domain Signal Waveform

Figure 3.1 and 3.2 are the time and frequency – domain pre-fault DFT voltage signal waveform obtained by plotting the V_n against time t and frequency respectively. The waveform is not sinusoidal even at No fault condition. It also shows that the maximum amplitude of V_n is 13.8pu at time of 2secs and largest magnitude of 275pu at 0.48Hz.

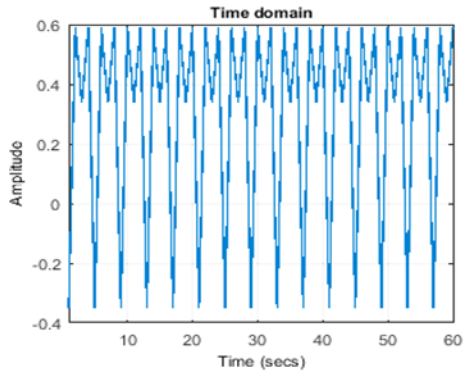

Figure 3.3: Mathematical Approach Pre-Fault DFT Current (In) t – domain Signal Waveform

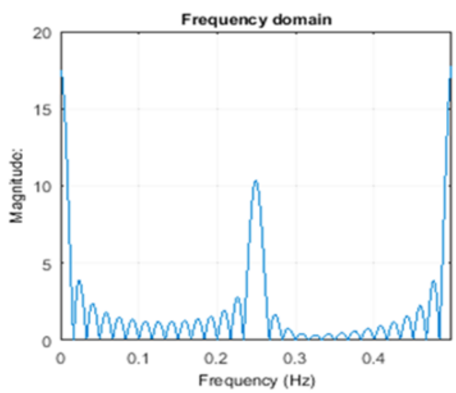

Figure 3.4: Mathematical Approach Pre-Fault DFT Current (In) s – domain Signal Waveform

Figure 3.3 and 3.4 are the time and frequency – domain pre-fault DFT current signal waveform obtained by plotting the I_n against time t and frequency respectively. The waveform is not sinusoidal even at No fault condition. It also shows that the maximum amplitude of I_n is 0.6pu at time of 2secs and largest magnitude of 11pu at 0.25Hz.

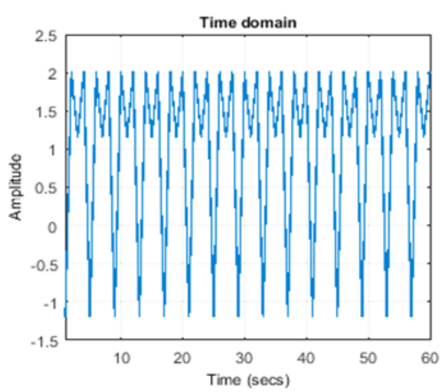

Figure 3.5: Mathematical Approach Three Phase Fault DFT Voltage (Vn) t – domain Signal Waveform

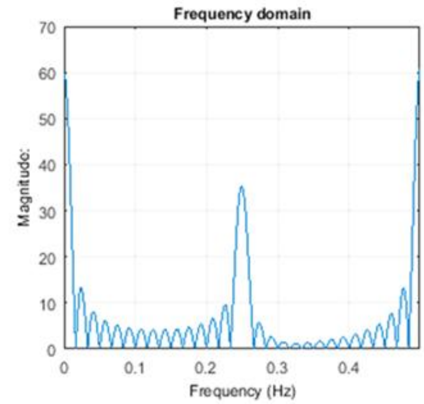

Figure 3.6: Mathematical Approach Three Phase Fault DFT Voltage (Vn) s – domain Signal Waveform

Figure 3.5 and 3.6 are the time and frequency domain three phase fault DFT voltage signal waveform obtained by plotting the V_n against time t and frequency respectively. The waveform is under three phase fault condition. It also shows that the maximum amplitude of V_n is 2pu at time of 2secs and largest magnitude of 35pu at 0.25Hz

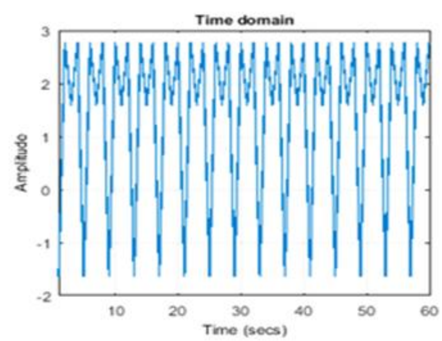

Figure 3.7: Mathematical Approach Three Phase Fault DFT Current (In) t – domain Signal Waveform

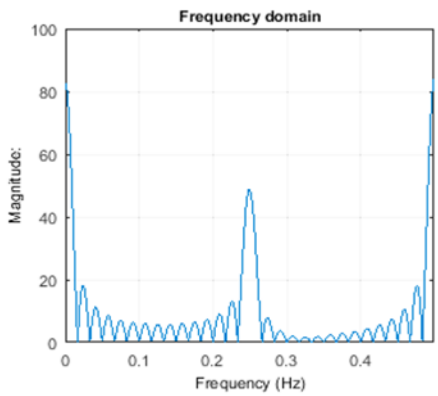

Figure 3.8: Mathematical Approach Three Phase Fault DFT Current (In) s – domain Signal Waveform

Figure 3.7 and 3.8 are the time and frequency – domain three phase fault DFT current signal waveform obtained by plotting the I_n against time t and frequency respectively. It also shows that the maximum amplitude of I_n is 2.8pu at time of 2secs and largest magnitude of 50pu at 0.25Hz.

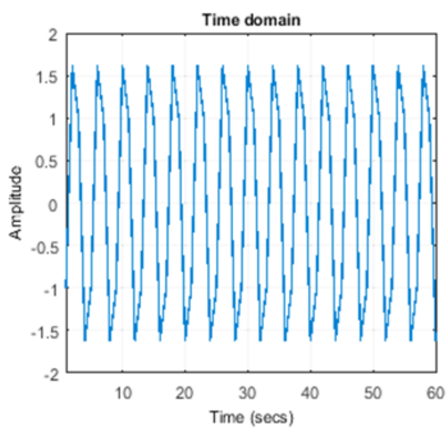

Figure 3.9: Mathematical Approach Pre-Fault FFT Voltage V(k) t – domain Signal Waveform

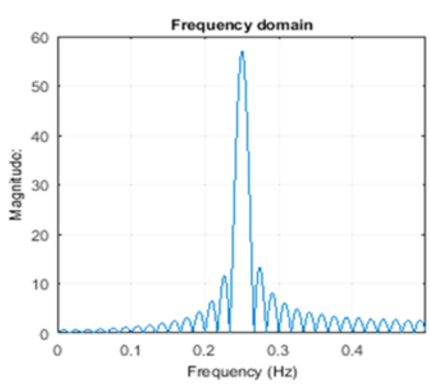

Figure 3.10: Mathematical Approach Pre-Fault FFT Voltage V(k) s – domain Signal Waveform

Figure 3.9 and 3.10 are the time and frequency – domain pre-fault FFT voltage signal waveform obtained by plotting the V_k against time t and frequency respectively. The waveform is not sinusoidal even at No fault condition. It also shows that the maximum amplitude of V_k is 1.6pu at time of 2secs and largest magnitude of 55pu at 0.25Hz.

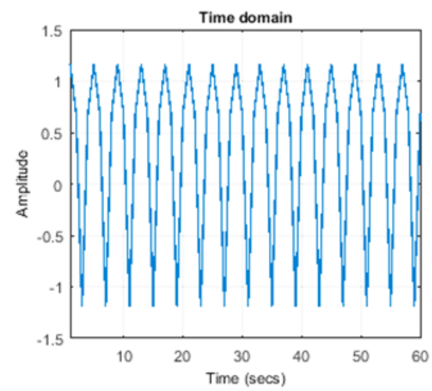

Figure 3.11: Mathematical Approach Pre-Fault FFT Current I(k) t – domain Signal Waveform

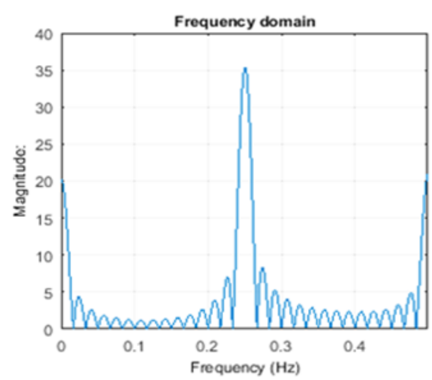

Figure 3.12: Mathematical Approach Pre-Fault FFT Current I(k) s – domain Signal Waveform

Figure 3.11and 3.12 are the time and frequency – domain pre-fault FFT voltage signal waveform obtained by plotting the I_k against time t and frequency respectively. The waveform is not sinusoidal even at No fault condition. It also shows that the maximum amplitude of I_k is 1.2pu at time of 2secs and largest magnitude of 36pu at 0.25Hz.

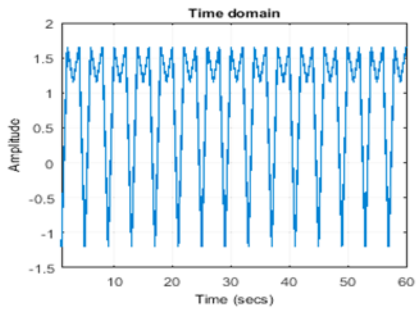

Figure 3.13: Mathematical Approach Three Phase Fault FFT Voltage (V_k) t – domain Signal Waveform

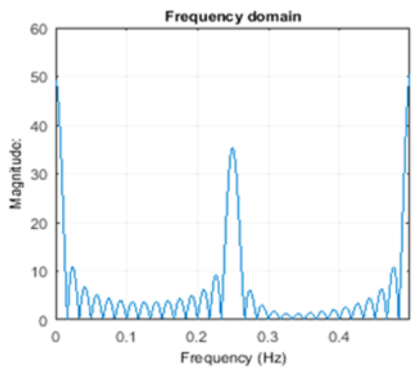

Figure 3.14: Mathematical Approach Three Phase Fault FFT Voltage (V_k) s – domain Signal Waveform

Figure 3.13 and 3.14 are the time and frequency – domain three phase fault FFT voltage signal waveform obtained by plotting the V_k against time t and frequency respectively. The waveform is under three phase fault condition. It also shows that the maximum amplitude of V_k is 1.7pu at time of 2secs and largest magnitude of 35pu at 0.25Hz.

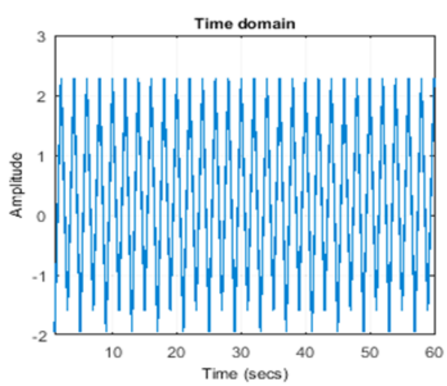

Figure 3.15: Mathematical Approach Three Phase Fault FFT Current I(k) t – domain Signal Waveform

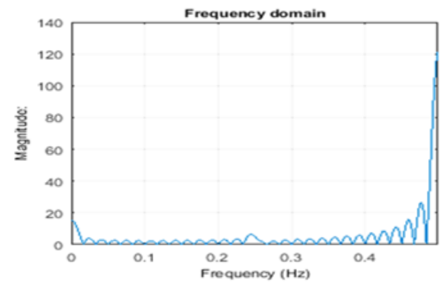

Figure 3.16: Mathematical Approach Three Phase Fault FFT Current I(k) s – domain Signal Waveform

Figure 3.15 and 3.16 are the time and frequency – domain three phase fault FFT current signal waveform obtained by plotting the I_K against time t and frequency respectively. It also shows that the maximum amplitude of I_K is 2.2pu at time of 2secs and largest magnitude of 10pu at 0.25Hz.

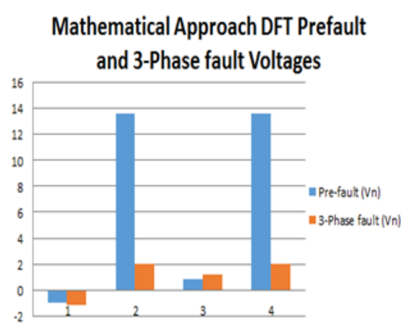

Figure 3.17: Mathematical Approach DFT Pre-faultand Three Phase fault Voltages

Figure 3.17 show that pre-fault voltage is higher than that of three phase fault by 10.8pu. This is because of the occurrence of fault.

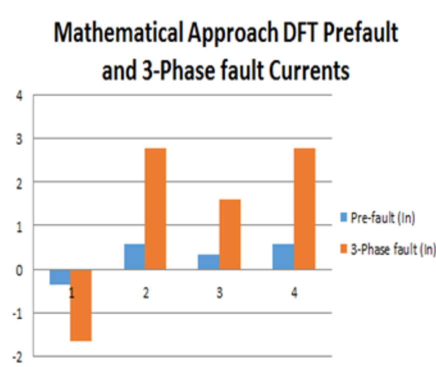

Figure 3.18: Mathematical Approach DFTPre-fault and Three Phase fault Currents

Here, the three phase fault current is higher than that of pre-fault by 2.5pu. This corresponds to the characteristics of fault, which is that, if fault occurs on the transmission line, the line current increases while the voltage will be reduced.

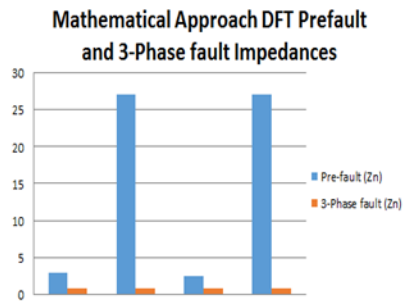

Figure 3.19: Mathematical Approach DFTPre-fault and Three Phase fault Impedances

Since the impedance of three phase fault is reduced to about 0.1pu from 27pu that means the presence of fault reduces the impedance, voltage and increase the current.

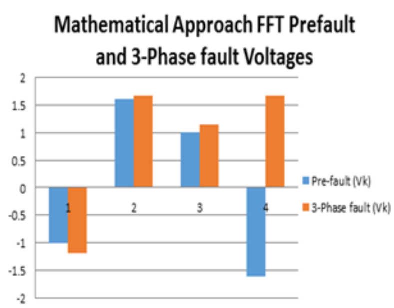

Figure 3.20: Mathematical Approach FFTPre-fault and Three Phase fault Voltages

Figure 3.20 show that three phase fault voltage is higher than that of pre-fault by a negligible value of about 0.1pu. This is not in conformity with the fault characteristics.

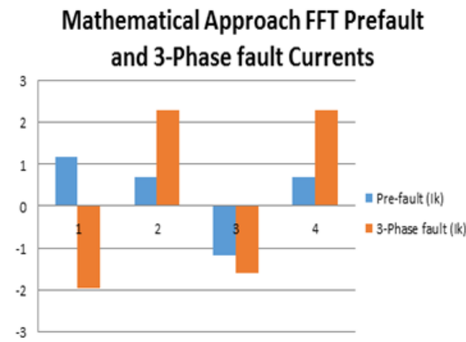

Figure 3.21: Mathematical Approach FFTPre-fault and Three Phase fault Currents

Here, the three phase fault current is higher than that of pre-fault by 1.2pu. This corresponds to the characteristics of fault, such that, under fault condition, the transmission line current increases while the voltage will be reduced.

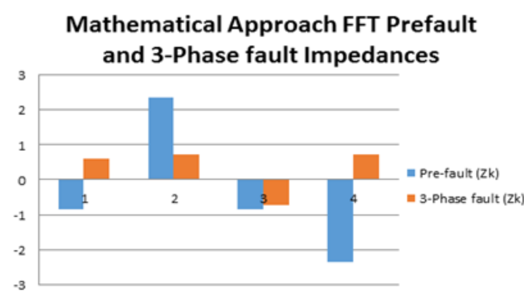

Figure 3.22: Mathematical Approach FFTPre-fault and Three Phase fault Impedances

The impedance of three phase fault is reduced to about 0.5pu from 2.2 pu, that means the presence of fault reduces the impedance, voltage and increase the current.

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