Some Real-Life Applications of Dynamical Systems

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Abstract -- In this paper, we discuss some applications of exciting fields like Mathematics, Statistics, Medical, Biological Sciences, Engineering, Economics etc. We show their real-life applications both theoretically and analytically considering some phenomena of the nature. We mainly focused on some applications of dynamical system in real life cases known as chaos, iteration, fractal geometry, Mandelbrot and Julia sets. We derive some mathematical formula concerning dynamical systems. Necessary programs are considered for all cases. We use Mathematica and MATLAB to perform programming.

Indexed Words: Chaos, Iteration, Fractal, Mandelbrot and Julia sets.

I. INTRODUCTION

Dynamical system has its application in almost all area of science, engineering and social science. We give some general ideas of dynamical systems and chaotic dynamical systems with three classical examples of dynamical systems in chemistry, biology and physics. We discuss about four tool kits of dynamical activities known as chaos, iteration, fractal with Mandelbrot and Julia sets. These represent a popular and exciting field of mathematics developed in recent decades. We discuss some applications of dynamical systems with special focus on the said four areas. In the concluding section, we give an example of dynamical systems from finance and ecology. Finally, we give some mathematical examples of chaotic and non-chaotic dynamical systems.

II. METHODOLOGY

There are some of senses to analyze dynamical systems and its applications. In our research we tried to show some applications of dynamical systems considering different and suitable functions and some phenomena of nature. We use some mathematical software's like MATHEMATICA, MATLAB etc. so that we can describe the graphical representation of our mathematical research.

III. MATHEMATICAL PRELIMINARIES

We need some basic definitions to be used elsewhere of this article. We mention these definitions as follows:

Basic definitions:

Dynamical System:

A dynamical system is a state space S , a set of times T and a rule R for evolution, $R: S\times T\rightarrow S$ that gives the consequent(s) to a state $s \in S$. A dynamical system can be a model describing the temporal evolution of a system.

Iteration:

Iterate means to repeat a process over and over again. To iterate a function means to evaluate the function over and over, using the output of the previous application as the input for the next. We write this as follows:

For a function $f(x)$,$f2(x)$ is the second iterate of f,namely $f(f(x))$,$f3(x)$ is the third iterate $f(f(f(x)))$, and in general,$fn(x)=f\circ f\circ f\circ f\circ\ldots\ldots\ldots\circ f(x)$ is the n-fold composition of with itself.

For Example, if $f(x)=2x+1$, then $f2(x) = f(f(x))=f(2x+1)=2(2x+1)+1$,

$f3(x)=f(f2(x))=f(4x+3)=2(4x+3)+1$

Thus we can determine $fn(x)$, Note that $fn(x)$ does not mean the nth power of $f(x)$, rather it means the n th iterate of f .

Chaos:

In common usage, "chaos" means "a state of disorder". However, in chaos theory, the term is defined more precisely. Although no universally accepted mathematical definition of chaos exists, a commonly used definition originally formulated by Robert L. Devaney says that, for a dynamical system to be classified as chaotic, it must have these properties:

- it must be sensitive to initial conditions
- it must be topologically mixing
- it must have dense periodic orbits

rated at x.

Fractals:

A fractal is a rough or fragmented geometric shape that can be subdivided in parts, each of which is a reduced size of copy of whole. i.e. they are from class of self-similar objects.

Benoît Mandelbrot observed that fractals are sets with non-integer dimensions and are found everywhere in nature. He observed that the proper estimation of most rough shapes that we see around us is not similar to smooth shapes, but similar to fractal idealized shapes. He said “clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line”.

Some of these mathematical structures are Sierpinski’s gasket, Cantor comb, Von Koch’s snowflakes and many in the list. Fractals describe many real-world objects like mountains, clouds, lightening, coastline and many more.

Mandelbrot and Julia Set:

The Mandelbrot set is the two-dimensional subset of this set that results if we always pick $z_0=0$ and allow c to vary. (The mathematical definition is rather more complex, than this, but it turns out to be equivalent).

The Julia set for a complex number k is the two-dimensional set that results if we always pick $c = k$ and allow z_0 varying.

IV. EXPERIMENTS AND RESULTS

Dynamical system has its application in almost all area of science, engineering and social science.

At first, we discuss some applications of dynamical systems in different sectors like:

Mathematics & Statistics: Using very simple dynamical systems, mathematicians discovered predictable and unpredictable phenomena. Using the histogram to display the data comprising orbits proves a connection to statistics.

Physics, Chemistry & Biology: The motions of the simple pendulum, the changes chemical undergo and the rise and fall of populations are three classical examples of dynamical systems in physics, chemistry & biology respectively.

Business and Banking: In business and banking, dynamical system is used to calculate interest due on saving balances or loans. It is used to predict in stock-market fluctuations.

Biology: In biology, dynamical system is used to predict growth and decline of population. The basic process of cell division is an example of dynamical system.

Computer science: In computer science, many of algorithms that computer scientist use to solve equations involved or dynamical system.

Economics: In economics, dynamical system is used to predict how the economy will change.

Ecology: In ecology, dynamical system is used to study the growth & decline of populations.

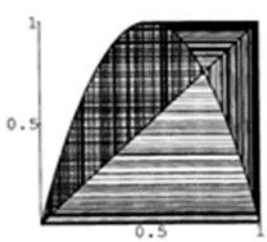
Medical science: In medical science, dynamical system is used to predict medicine absorption by the body.

Meteorology: In meteorology, dynamical system is used to predict long-term weather change.

Chaos: The phenomena called chaos is one of the most interesting scientific discoveries of recent years. Much of what happens in nature can be modeled by mathematical equations. For years, scientists have been developing mathematical models for everything from the motion of the simple pendulum to the motion of planets in the solar system. There are equations for the rise and fall of populations and the ups and down of the economy. Every field of science and engineering features its own typical mathematical models.

For example, consider the weather. It still seems impossible to accurately forecast the weather one week ahead of time, despite the fact that we can know the current weather at virtually every point of the globe at any given moment. For years scientists thought that if only they could have access to bigger and better machines, or to faster algorithms, or to more initial data, then they would be able to make accurate predictions. However, in the last 25 years scientists and mathematicians have come to realize that that can never be the case. The culprit is mathematical phenomena known as chaos. When chaos is a part of mathematical a model, faster and more accurate computing will never lead to complete predictability, for chaos means that very small changes in the initial configuration of the system may lead to great discrepancies down the road. Called the 'Butterfly Effect', this phenomenon accounts for our inability to make accurate predictions in the weather, for example, despite enormous computing power & loads of data. Indeed, as we shall see below, a mathematical expression as simple as 4x(1-x), when thought of as a dynamical system, can be incredibly chaotic. To understand this statement, we will have to gather all of our mathematical and computational tools.

A Glimpse of Chaotic Behaviour



Graphical Analysis of F(X)=4X(1-X)

Several Chaotic Effects: Chaotic phenomena abound in the sciences, they can be found in nearly all branches of non-linear modeling. In chemistry and theoretical biology, where interactions between various components, chemical elements or population densities, give rise to non-linear equations. Chaotic effects have been traced as a superposition on rhythmic activity of the heart and the brain. Mathematical biologists have been successful in explaining unexpected fluctuations in the incidence of children's diseases by chaos theory. Research is going on this direction in economic and monetary modeling, even in fields which until now hardly used mathematics.

Considering different and suitable functions, we have eventually generated some beautiful and natural images. These images result come from known and famous mathematical functions or combination of them. On the basis of MATHEMATICA and MATLAB Program we present here some real life applications of said four tool kits of dynamical activities.

Applications of Iteration:

Generating Tiles:

Function: f(x,y)=sin(x^3+y^3)

Number of Iteration:30

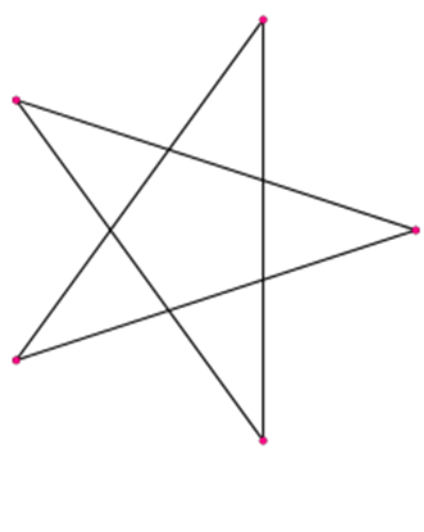
A tile type of a finite subdivision rule associated to the right-angled cuboctahedron after repeated subdivision.

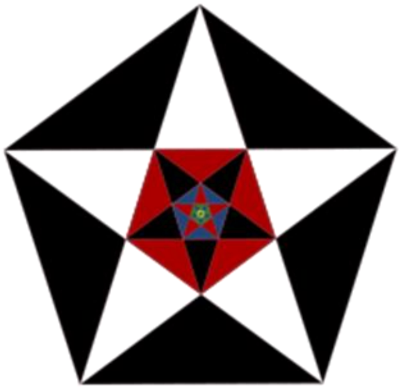
Generating Pentagram Diagram:

Function:

$$\varphi = 1 + 2\sin(\pi/10) = 1 + 2\sin 18^\circ$$

Number of Iteration: 50 , n= 10





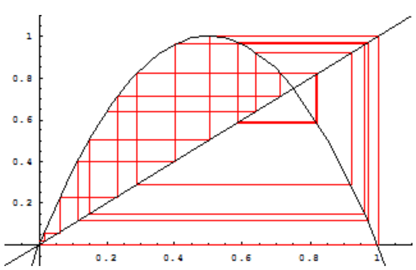
n=50

Pentagram diagram with vector iteration

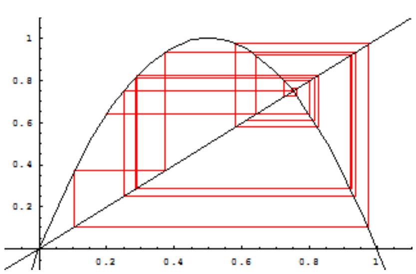
Applications of Chaos:

If we look at the logistic iteration rule x→ 4x(1-x) with two nearby seeds (0.2 and 0.2001) the orbit of seed 0.2 is closed to that of seed 0.2001 for fast 13 or 50 iteration. After that they move away very differently. This phenomenon is called sensitivity to initial conditions. Sensitivity to initial condition means that the orbits of to nearby seeds behave very differently after some iteration.

After 20 iterations:



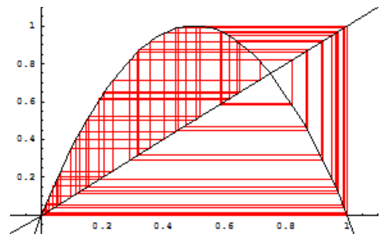
x = 0.590364(when x0 = 0.2)



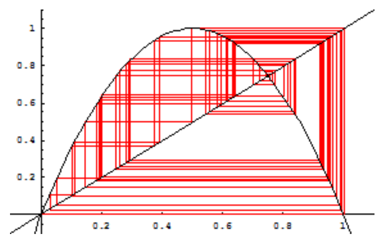
x = 0.61174(when x0 = 0.2001)

Difference = 0.021376

After 52 iterations:



x = 0.615035(when x0 = 0.2)



x = 0.999999 (when x0 = 0.2001)

Difference = 0.384964

So we see that, although the orbit of points for initial seeds 0.2 & 0.2001 do not differ for first few iteration, but it tremendously change after that. Similarly, in case weather, we may predict it for next few days, but the condition and properties of weather change suddenly. So long time prediction is not possible yet. But we are researching for the better procedure to predict the chaos for longer time.

Chaos in Medical Science

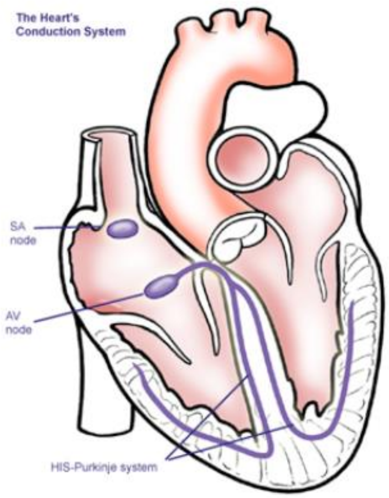
The heart has its own pacemaker.

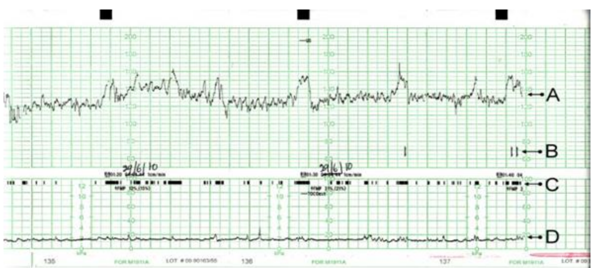
The heart's cycle feeds back to its pacemaker keeping a regular rhythm.If this rhythm becomes chaotic, it leads to arrhythmia, which can lead to cardiac arrest and to death. Frequency-locking can be seen in the circle map

(n+1)! = !n + k sin(n!)

L. Glass proposed a model for the frequency locking in the heart.

The theory agrees well with experiments on cells from a chicken heart.





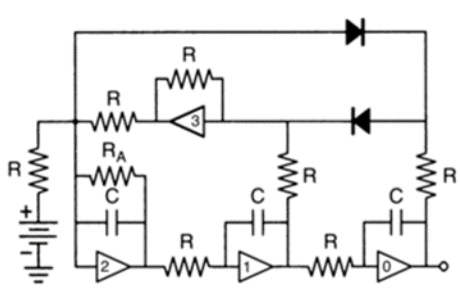
Chaos in Mechanics

A jerk system's behavior is described by a jerk equation, and for certain jerk equations, simple electronic circuits may be designed which model the solutions to this equation. These circuits are known as jerk circuits.

An example of a jerk equation with nonlinearity in the magnitude of x is:

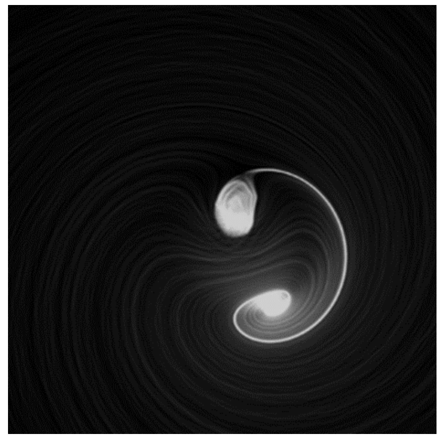
$$\frac{d^3 x}{dt^3} + A\frac{d^2 x}{dt^2} + \frac{dx}{dt} - |x| + 1 = 0.$$

Here, A is an adjustable parameter. This equation has a chaotic solution for A=3/5 and can be implemented with the following jerk circuit; the required nonlinearity is brought about by the two diodes:



In the above circuit, all resistors are of equal value, except $R_A=R/A=5R/3$, and all capacitors are of equal size. The dominant frequency will be $1/2\pi RC$. The output of op amp 0 will correspond to the x variable, the output of 1 will correspond to the first derivative of x and the output of 2 will correspond to the second derivative.

Chaos In Mathematics And Physics (Ikeda Map):



The trajectories of 2000 random points in an Ikeda map with u = 0.918.

In physics and mathematics, the Ikeda map is a discrete-time dynamical system given by the complex map

$$z_{n+1} = A + B z_n e^{i(|z_n|^2 + c)}$$

The original map was proposed first by Ikeda as a model of light going around across a nonlinear optical resonator (ring cavity containing a nonlinear dielectric medium) in a more general form. It is reduced to the above simplified "normal" form by Ikeda, Daido and Akimoto stands for the electric field inside the resonator at the n-th step of rotation in the resonator, and A and C are parameters which indicates laser light applied from the outside, and linear phase across the resonator, respectively. In particular the parameter $B \leq 1$ is called dissipation parameter characterizing the loss of resonator, and in the limit of B=1 the Ikeda map becomes a conservative map.

The original Ikeda map is often used in another modified form in order to take the saturation effect of nonlinear dielectric medium into account:

$z_{n+1}=A+Bz_n e^{i(|z_n|^2+c)}$

A 2D real example of the above form is:

$x_{n+1}=1+u(x_n \cos t_n - y_n \sin t_n)$

$y_{n+1}=u(x_n \sin t_n + y_n \cos t_n)$

where u is a parameter and

$t_n=0.4-6/(1+x_n^2+y_n^2)$.

For $u \geq 0.6$, this system has a chaotic attractor.

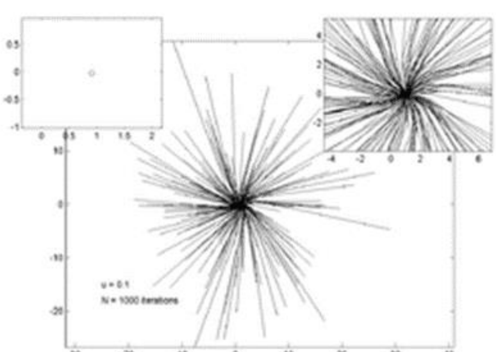
Attractor

This animation shows how the attractor of the system changes as the parameter is varied from 0.0 to 1.0 in steps of 0.01. The Ikeda dynamical system is simulated for 500 steps, starting from 20000 randomly placed starting points. The last 20 points of each trajectory are plotted to depict the attractor. Note the bifurcation of attractor points as is increased.

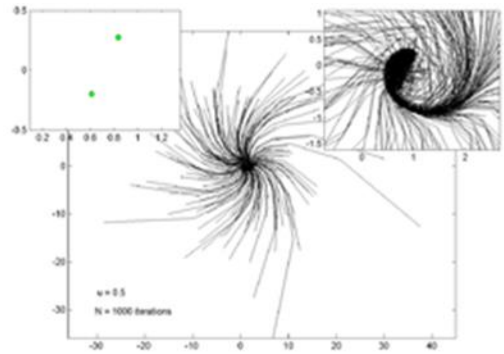
$u = 0.3 u = 0.5$

Point trajectories

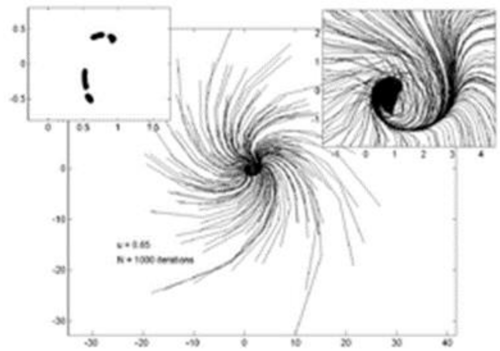
The plots below show trajectories of 200 random points for various values of . The inset plot on the left shows an estimate of the attractor while the inset on the right shows a zoomed in view of the main trajectory plot.



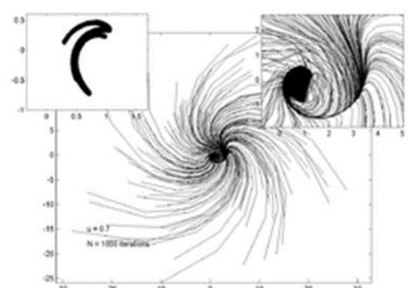
u = 0.1



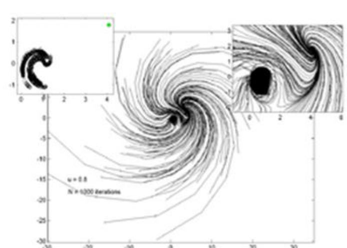
u = 0.5



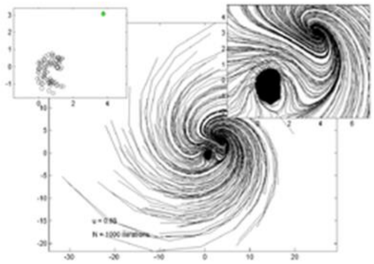
u = 0.65



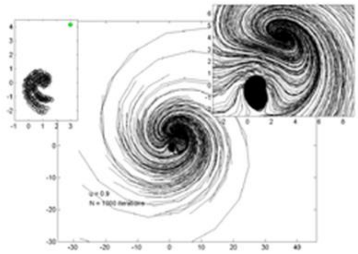
u = 0.7



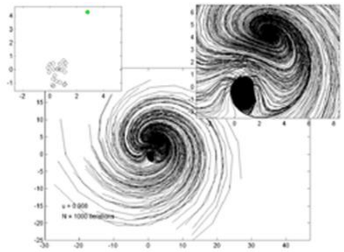
u = 0.8



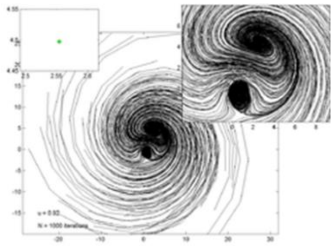
u = 0.85



u = 0.9



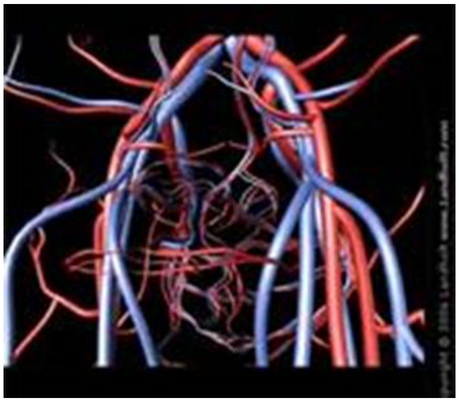
u = 0.908



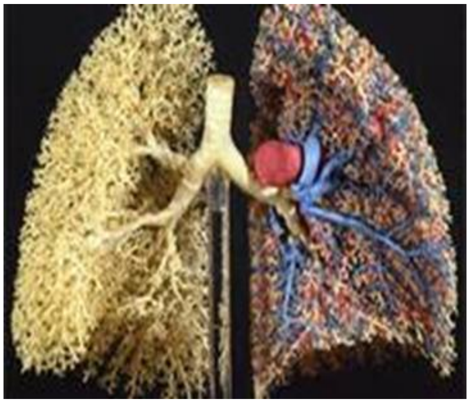
u = 0.92

Applications Of Fractal Geometry On Human Anatomy:

Fractals have tremendous application on human anatomy,molecules, astrophysics, telecommunication, military ,bacteria culture and many more. In human anatomy our artery system, brain, membrane ect., all are divided into self-similar parts and all have fractal dimension.



Arteries



Bronchial Tubes

Anatomical Structure	Fractal Dimension
Bronchial Tubes	very close to 3
Arteries	2.7
Brain	2.73 – 2.79
Alveolar Membrane	2.17

Source:library.thinkquest.org

On Telecommunication Antenna:

In telecommunication fractal antenna had bought big revolution. Designing antenna for narrow range frequency was a big problem for portable antenna used in cellular phone because frequency smaller than quarter length is not efficient. Also multiband antenna was bigger in size.

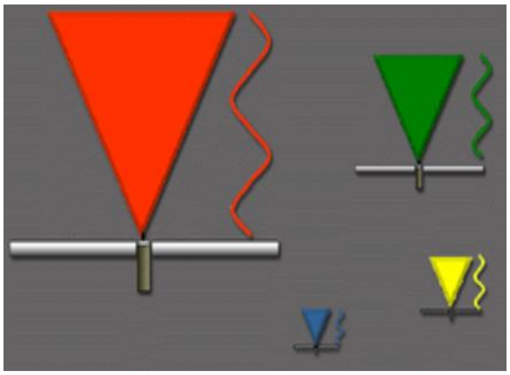


Fig1: Four separate antenna

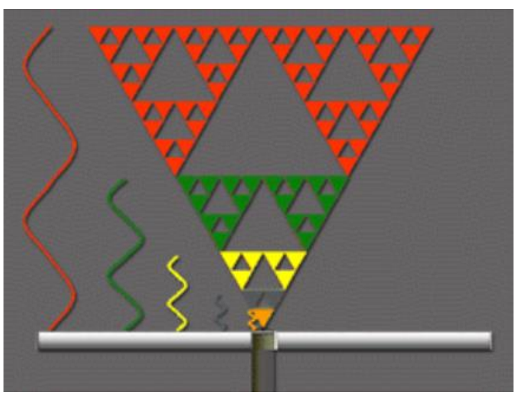


Fig 2: One antenna for four separate band

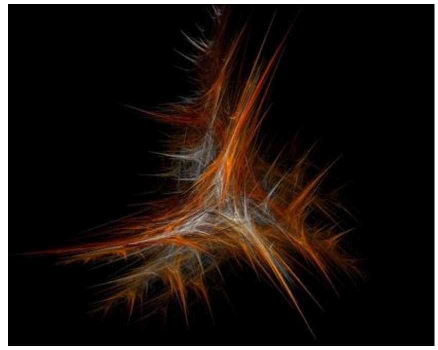
Antenna built with small number of iteration can exhibit sensitivity at several frequencies and if number of iteration increases the lowest frequency gets lower and addition higher frequencies are added.

The triangular shaped fractal, Sierpinki's gasket with similarity dimension 1.58(log 3/ log 2), as discussed earlier, is common study among researchers in fractal antenna [Puente, 2000]. Formulation of relationship between fracatal dimension and antenna directivity had been established by Douglas H. Werner, Randy L. Haut,and Pingjuan L. WerneJ .

Generating Flame:

Function: f(x,y)= 〖cos〗^2 x+ 〖cos〗^2 y

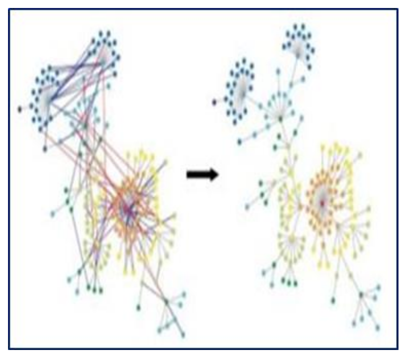
Number Of Iteration: 90



Generating Skeleton of a Network:

Function: (z)=z^2 +c,c=-1.3+0.1i

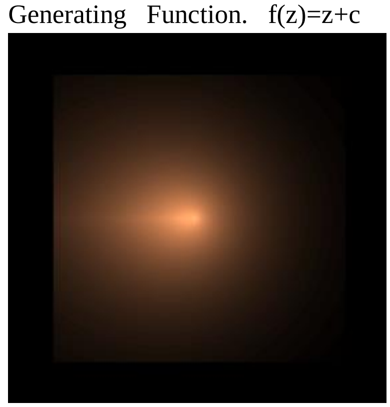
Number Of Iteration:20



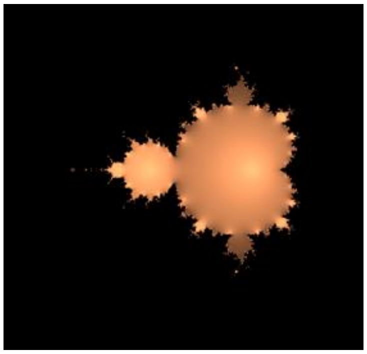
Skeleton of a Network

Applications Of Mandelbrot And Julia Sets

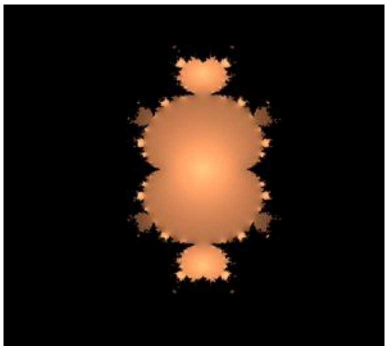
Generating Mandelbrot set:
Generating Function. f(z)=z+c c=-.6+0i



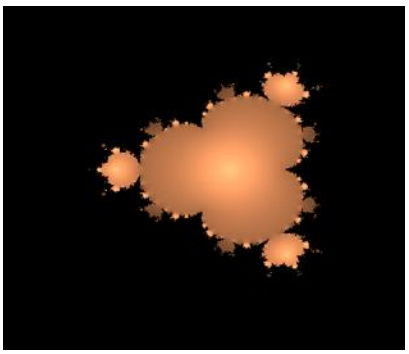
Generating Function: f(z)=z2+c , c=-.6+0i



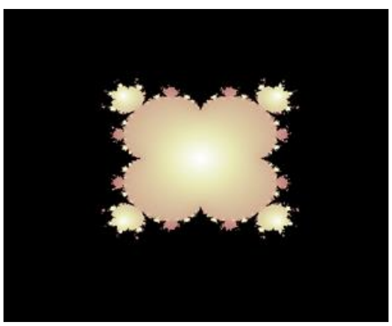
Generating function: f(z)= z3 +c, c=.6+0i



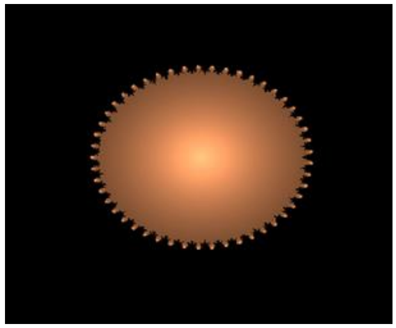
Generating Function: f(z)=z4+c, c=-.6+0i



Generating Function: f(z)=z5+c, c=-.6+0i



Generating Function: f(z)=z50+c, c = -.2+0i



Generating Julia set:

Generating Function: f(z) = z2+ c where c = 0.27+0.53i.

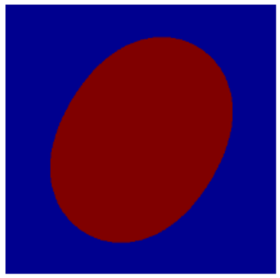


Fig. 1st iteration (when z = 0)

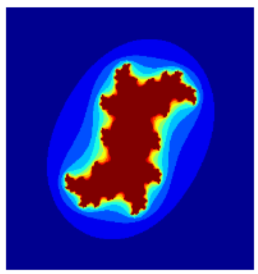


Fig. 10th iteration

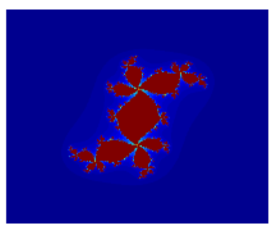


Fig. 50th iteration

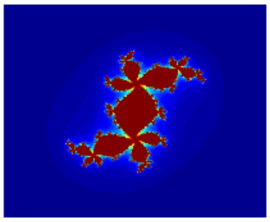
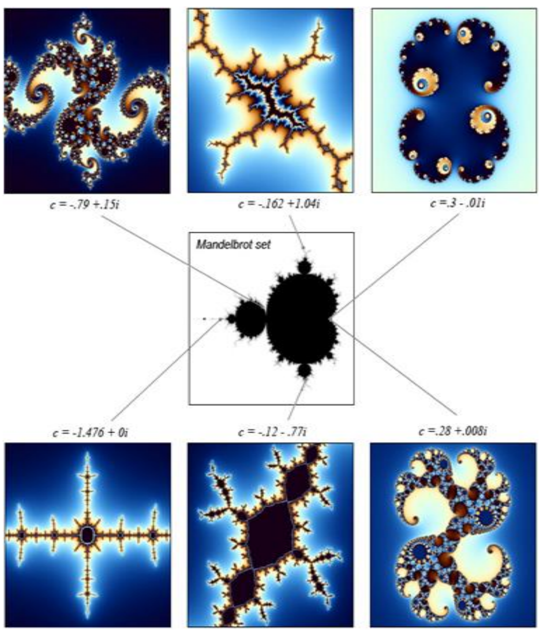


Fig. 120th iteration: Complete Julia set

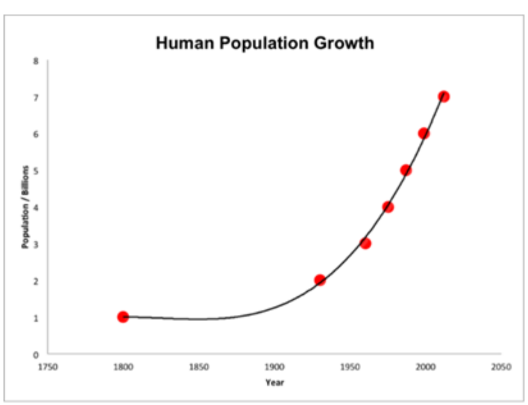
Generated Mandelbrot Set

Mandelbrot set



Applications of Nonlinear Iteration To Ecology

Human Population Growth in a Are



The human population has been growing rapidly

Human Population Growth in Cont

Estimated and projected populations of the world and its continents

V. CONCLUSION

In this article, we discuss some Applications of Dynamical Systems in Real Life considering some popular fields like Mathematics, Statistics, Medical, Biological Sciences, Engineering, Economics etc. we mainly focus to determine some applications of dynamical system in real life cases such as Iteration, Chaos, Mandelbrot & Julia sets and Fractals.

We show their real-life applications both theoretically and analytically considering sophenomenon from the nature. We analyze the results of some cases by making suitable program using MATHEMATICA and MATLAB programming.

In the future, we will make suitable programs for those functions which are considered here. We will concentrate to specify some areas already discussed here so that we can establish their mathematical formulas concerning Dynamical Systems. Special emphasize will be given on chaotic phenomena and mathematical models.

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REFERENCES

[1] P. Ahmed, Chaotic homeomorphisms in one-dimensional dynamics, Proc., Iwate Univ., Japan, May 5-6, 2001.

[2] R. L. Devaney, An Introduction to Chaotic Dynamical Systems, MenloPark: Benjamin/Cummings, 1986.

[3] R. L. Devaney, Fractals: A Tool Kit of Dynamics Activities, Key Curriculum Press, 1999.

[4] Eugene Don, Schaum’s Outline of Theory and Problems of Mathematica, McGRAW-HILL, 2001.

[5] Willam Mayo, Martin Cwiakala, Schaum’s Outline of Programming With Fortran, September 1994.

[6] B. Hasselblatt and A. Katok, Introduction to the Modern Theory of Dynamical Systems, Cambridge University Press, (1995, 1997).

[7] Morris W. Hirsch, Stephen Smale and Robert Devaney. Differential Equations, Dynamical Systems, and An Introduction to Chaos, ISBN 0-12-349550-4, Academic Press, 1974.

[8] Hazewinkel, Michiel, ed. (2001), "Chaos", Encyclopedia of Mathematics, Springer, ISBN 978-1-55608-010-4

[9] Nonlinear Dynamics. Models of bifurcation and chaos by Elmer G. Wiens