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Minimum Inverse Dominating Energy of a Graph

KAVITA PERMI [1], K. N. PRAKASHA [2]
[1] *Department of Mathematics, Presidency University, Bengaluru-560064, India.*
[2] *Department, Vidyavardhaka College of Engineering, Mysuru- 570002, India.*

Abstract- The Partition Laplacian energy of a graph was introduced by K. N Prakasha. In this paper, by the motivation of this energy, the Minimum Inverse Dominating energy $E_{D'}$ (G) of a graph is introduced and the $E_{D'}$ (G) of some important graph classes is discussed.

Key Words- minimum inverse dominating set, minimum inverse dominating matrix, minimum inverse dominating eigen values, minimum inverse dominating energy of a graph.

I. INTRODUCTION

For standard definitions and terminology regarding graph theory, we refer [2]. Throughout this paper, we consider simple, undirected graphs without loops and multiple edges. No signs and marks are on the vertices and edges. The concept of graph energy was introduced by I.Gutman [1] as the sum of the absolute values of the eigenvalues of the adjacency matrix of the given graph G. One can find other types of energy such as distance energy, maximum degree energy, color energy.

In 21st century, Energy of graph and Topological indices attracting the researchers due to their wide range of applications. For the estimation of total π-electron energy of a molecule we can use these concepts which are having great importance in Chemistry.

II. THE MINIMUM INVERSE DOMINATING ENERGY

The inverse domination in graphs was introduced by V. R. Kulli and S.C.Sigarkanti in [4] in 1991.
Let G be a simple graph of order n with vertex set V = $\{v_1, v_2, \ldots, v_n\}$. and edge set E. A subset D of V is called a dominating set of G if every vertex of V - D is adjacent to some vertex in D. Let D be a minimum dominating set of G. If V-D contains a dominating set say D' of G, then D' is called an inverse dominating set with respect to D.

Any inverse dominating set with minimum cardinality is called a minimum inverse dominating set. Let D′ be a minimum inverse dominating set of a graph G. The minimum inverse dominating matrix of G is the n × n matrix defined by $A_{D'}$ (G) = (aij), where

$$a_{ij} = \begin{cases} 1 \ if \ v_i v_j \in E, \\ 1 \ if \ i = j \ and \ v_i \in D', \\ 0 \ otherwise \end{cases}$$

The characteristic polynomial of $A_{D'}$(G) is denoted by $fn\,(G, \lambda) = \ det\big(\lambda I \ - \mathrm{AD}'(\mathrm{G})\big)$
The minimum inverse dominating eigenvalues of the graph G are the eigen-values of $A_{D'}$(G). Since $A_{D'}$(G) is real and symmetric, its eigenvalues are real numbers and we label them in non-increasing order $\lambda'_1 \geq \lambda'_2 \geq \cdots \geq \lambda'_n$. Theminimum inverse dominating energy of G is defined as
$E_D{}'$ (G) $=\sum_{i=1}^{n} |\lambda_i|$
Note that the trace of $A_{D'}$(G) = Domination Number = k.

III. MINIMUM INVERSE DOMINATING ENERGY OF SOME STANDARD GRAPHS

Theorem 4.1. If K_n is the complete graph of order n, then

$$A_{D'}(K_n) = (n - 2) + \sqrt{n^2 - 2n + 5}.$$

Proof. Let K_n be the complete graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$. The minimum inverse dominating set is D = $\{v_1\}$. Then

$$A_{D'}(K_n) = \begin{bmatrix} 1 & 1 & 1 & & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & & 1 & 1 \\ & \vdots & & \ddots & & \vdots \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{bmatrix}$$

The characteristic equation is

$$(\lambda+1)^{n-2}[\lambda^2-(n-1)\lambda-1]=0.$$

and the minimum inverse dominating eigenvalues are

$A_{D'}\operatorname{spec}(K_n)$ =

$$\begin{pmatrix} -1 & \frac{(n-1)-\sqrt{n^2-2n+5}}{2} & \frac{(n-1)+\sqrt{n^2-2n+5}}{2} \\ n-2 & 1 & 1 \end{pmatrix}$$

Therefore the minimum inverse dominating energy is

$$A_{D'}(K_n) = (n-2)+\sqrt{n^2-2n+5}.$$

Theorem 4.2: The minimum inverse dominating energy of the star graph $K_{1,n-1}$ is

$A_{D'}(K_{1,n-1}) = (n-2)+\sqrt{4n-3}$

Proof: Let $K_{1,n-1}$ be the star graph with vertex set $V = \{v_0, v_1, v_2, \ldots, v_{n-1}\}$ and the minimum inverse dominating set is $D' = \{v_1, v_2, \ldots, v_{n-1}\}$.

Then the minimum inverse dominating matrix is

$$A_{D'}(K_{1,n-1}) = \begin{bmatrix} 0 & 1 & 1 & & 1 & 1 \\ 1 & 1 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 1 & & 0 & 0 \\ & \vdots & & \ddots & & \vdots \\ 1 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

The characteristic equation is

$$(\lambda-1)^{n-2}[\lambda^2-\lambda-(n-1)]=0.$$

and the minimum inverse dominating eigenvalues are

$A_{D'}\operatorname{spec}(K_{1,n-1})$ =

$$\begin{pmatrix} 1 & \frac{1-\sqrt{4n-3}}{2} & \frac{1+\sqrt{4n-3}}{2} \\ n-2 & 1 & 1 \end{pmatrix}$$

Therefore the minimum inverse dominating energy is

$$A_{D'}(K_{1,n-1}) = (n-2)+\sqrt{4n-3}$$

Theorem 4.3: The minimum inverse dominating energy of the double star graph $S_{n,n}$ is

$A_{D'}(S_{n,n}) = (2n-4)+2\sqrt{n}+2\sqrt{n-1}$

Proof: **T**he minimum inverse dominating matrix is

$$A_{D'}(S_{n,n}) = \begin{bmatrix} 0 & 1 & 1 & & 1 & 1 & 0 & 0 & & 0 \\ 1 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 1 & & 0 & 0 & 0 & 0 & & 0 \\ & \vdots & & \ddots & \vdots & & \vdots & & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & & 0 & 0 & 1 & 1 & & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & & 0 & 1 & 0 & 1 & & 0 \\ & \vdots & & \ddots & \vdots & & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

The characteristic equation is

$$(\lambda-1)^{n-2}[\lambda^2-\lambda-(n-1)]=0.$$

and the minimum inverse dominating eigenvalues are

$A_{D'}\operatorname{spec}(S_{n,n})$ =

$$\begin{pmatrix} 1 & -\sqrt{n} & \sqrt{n} & 1-\sqrt{n-1} & 1+\sqrt{n+1} \\ 2n-4 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Therefore the minimum inverse dominating energy is

$$A_{D'}(S_{n,n}) = (2n-4)+2\sqrt{n}+2\sqrt{n-1}$$

Definition 4.4: The crown graph S_n^0 for an integer $n \geq 3$ is the graph with the vertex set $\{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$ and the edge set $\{u_i v_j : 1 \leq i, j \leq n, i \neq j\}$.

S_n^0 is therefore equivalent to the complete bipartitegraph $K_{n,n}$ with horizontal edges removed

Theorem 4.5 The minimum inverse dominating energy of crown graph S_n^0 is

$$A_{D'}(S_n^0) = 2(n-2) + \sqrt{n^2-2n+5} + \sqrt{n^2+2n-3}.$$

Proof: Let S_n^0 be the crown graph with vertex set $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$ and the minimum inverse dominating set is $D' = \{u_1, v_1\}$. Then the minimum inverse dominating matrix is

$$A_{D'}(S_n^0) = \begin{bmatrix} 1 & 0 & 0 & & 0 & 0 & 1 & 1 & & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 1 & \cdots & 1 \\ 0 & 0 & 0 & & 0 & 1 & 1 & 0 & & 1 \\ & \vdots & & \ddots & \vdots & & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 & 1 & 1 & \cdots & 0 \\ 0 & 1 & 1 & & 1 & 1 & 0 & 0 & & 0 \\ 1 & 0 & 1 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & & 1 & 0 & 0 & 0 & & 0 \\ & \vdots & & \ddots & \vdots & & \vdots & & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

The characteristic equation is

$$(\lambda-1)^{n-2}(\lambda+1)^{n-2}[\lambda^2-(n-1)\lambda-1][\lambda^2-(n-3)\lambda-(2n-3)] = 0.$$

and the minimum inverse dominating eigenvalues are

$$A_{D'}\,\mathrm{spec}(S_n^0) = \begin{pmatrix} 1 & -1 & \frac{(n-1)-\sqrt{n^2-2n+5}}{2} & \frac{(n-1)+\sqrt{n^2-2n+5}}{2} & \frac{(3-n)-\sqrt{n^2+2n-3}}{2} & \frac{(3-n)+\sqrt{n^2+2n-3}}{2} \\ n-2 & n-2 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Therefore the minimum inverse dominating energy is

$$A_{D'}(S_n^0) = 2(n-2) + \sqrt{n^2-2n+5} + \sqrt{n^2+2n-3}.$$

Theorem 4.6: The minimum inverse dominating energy of the Cocktail party graph $K_{n\times 2}$ is

$$A_{D'}(K_{n\times 2}) = (2n-3) + \sqrt{4n^2-4n-9}$$

Proof: Let $K_{n\times 2}$ be the star graph with vertex set $V = \bigcup_{i=1}^{n}\{u_i, v_j\}$ and the minimum inverse dominating set is $D' = \{u_1, v_1\}$.

Then the minimum inverse dominating matrix is

$$A_{D'}(K_{n\times 2}) = \begin{bmatrix} 1 & 0 & 1 & 1 & & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & & 1 & 1 & 1 & 1 \\ & \vdots & & & \ddots & & \vdots & & \\ 1 & 1 & 1 & 1 & & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & \cdots & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & & 1 & 1 & 0 & 0 \end{bmatrix}$$

The characteristic equation is

$$\lambda^{n-1}(\lambda+2)^{n-2}[\lambda^2-(2n-3)\lambda-2n] = 0.$$

and the minimum inverse dominating eigenvalues are

$$A_{D'}\,\mathrm{spec}(K_{n\times 2}) = \begin{pmatrix} 0 & -2 & \frac{(2n-3)-\sqrt{4n^2-4n+9}}{2} & \frac{(2n-3)+\sqrt{4n^2-4n+9}}{2} \\ n-1 & 1 & 1 & 1 \end{pmatrix}$$

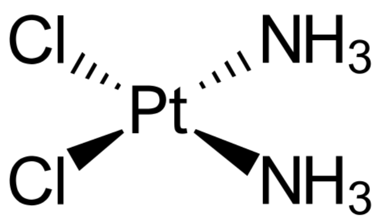
Therefore the minimum inverse dominating energy is

$$A_{D'}(K_{n\times 2}) = (2n-3) + \sqrt{4n^2-4n-9}.$$

Cancer is a disease which is caused by an uncontrolled division of abnormal cells in a part of the body. Now a days we can find many types of cancer like, Bladder cancer Lung cancer, Brain cancer, Melanoma, Breast cancer, Non-Hodgkin lymphoma, Cervical cancer, Ovarian cancer. Etc.,

Cisplatin is a medicine which is widely used against the Cancer disease. In this paper we are calculating the energy of the Cisplatin which is very much useful for further research and development in the treatment of Cancer.

Structural formula: ***$Pt(NH_3)_2Cl_2$***



ENERGY OF CISPLATIN

The inverse dominating matrix with the consideration of inverse dominating set {Pt} is given by

$$= \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Characteristic equation is

$$\lambda^3 \left(\lambda - \frac{\sqrt{17}+1}{2}\right) \left(\lambda + \frac{\sqrt{17}+1}{2}\right) = 0$$

and the minimum inverse dominating eigenvalues are

$$A_{D'}\,\mathrm{spec}(Cisplatin) = \begin{pmatrix} 0 & \frac{\sqrt{17}+1}{2} & \frac{-\sqrt{17}+1}{2} \\ 3 & 1 & 1 \end{pmatrix}$$

Therefore the minimum inverse dominating energy is

$$A_{D'}(Cisplatin) = \sqrt{17}.$$

REFERENCES

[1] I. Gutman, The energy of a graph, Ber. Math-Statist. Sekt. Forschungsz Graz, 103 (1978), 1-22.

[2] F. Harary, Graph Theory, Addison-Wesley Publishing Co., (1969).

[3] K. N. Prakasha, P Siva Kota Reddy, Ismail Naci Cangul, Partition Laplacian, Advanced Studies in Contemporary Mathematics, 27 (2017), No. 4, pp. 477 – 494.

[4] V. R. Kulli and S.C.Sigarkanti, Inverse domination in graphs, Nat. Acad. Sci . Letters, Vol. 14, 1991, 473-475.